Automorphic forms and automorphic L-functions

RIMS Conference

Organizers: Tomonori Moriyama (Osaka University)
Atsushi Ichino (Kyoto University)

Titles and Abstracts (As of January 9, 2012)

Jan.16 (Mon)

10:05-10:55 **Kazuki Morimoto** (Osaka City University)

Title: *On special values of tensor product L-functions of an inner form of* $\mathrm{G}Sp(4)$ *and* $\mathrm{G}L(2)$

Abstract: Let G be a similitude quaternion unitary group of degree two over a number field F whose semisimple F-rank is one. We note that G is an inner form of GSp(4). By using the restriction of the Siegel Eisenstein series on GU(3,3), we obtain an integral representation of $L\left(s, \Pi \otimes \sigma\right)$ where Π (resp. σ) is an irreducible cuspidal representation of $G\left(\mathbb{A}\right)$ (resp. $\mathrm{GL}_2\left(\mathbb{A}\right)$). As an application, when the base field is $\mathbb{Q}$, we prove the algebraicity of the special values of $L\left(s, \Pi \otimes \sigma\right)$ when both Π and σ correspond to the holomorphic cusp forms of the same weight $\ell > 6$.

11:10-12:10 **Taku Ishii** (Seikei University)

Title: *Archimedean zeta integrals for the exterior square L-functions on GL(n)*

Abstract: Bump and Friedberg discovered a zeta integral which interpolates the standard and the exterior square L-functions on GL(n). We compute the real archimedean zeta integral when the real component of automorphic representation is isomorphic to the principal series representation.

13:30-14:30 **Michitaka Miyauchi** (Kyoto University)

Title: *On local newforms for unramified* $\mathrm{U}(2,1)$

Abstract: In this talk, we consider Gelbart and Piatetski-Shapiro's integrals attached to generic representations π of the unramified unitary group in three variables defined over a p-adic field. We introduce the notion of newforms by using a certain family of compact

open subgroups, and show that the Whittaker function associated to a newform attains the L-factor of π defined through Gelbart and Piatetski-Shapiro's integral.

14:45-15:45 **Yoshi-hiro Ishikawa** (Okayama University)

Title: *On H-periods of generic cusp forms on* U(3)

Abstract: In early 90's, Gelbart and his collaborators investigated $U_{E/F}(2)$-periods $P_H(\phi)$ of cusp forms ϕ on $U_{E/F}(3)$. Through investigation of zeta integrals with trace formula results, they related $P_H(\phi)$ to a special value of L-function of $GL_2(E)$ up to bad factors. We would report some progress on bad factors, when cusp form ϕ is generic.

16:00-17:00 **Atsushi Murase** (Kyoto Sangyo University)

Title: *Borcherds lifts and symmetries*

Abstract: In this talk, we give a characterization of holomorphic Borcherds lifts on an orthogonal group $O(2, n+2)$ by certain symmetries. As an application we show that modular polynomials are characterized by symmetries.

Jan. 17 (Tues)

9:45-10:45 **Atomu Otsuka** (Kyoto University)

Title: *On the image of the Saito-Kurokawa lifting over a totally real number field and the Maass relation*

Abstract: The space of Siegel modular forms generated by the Saito-Kurokawa lifting is known to satisfy the relation between the Fourier coefficients, called the Maass relation. We introduce a similar phenomenon holds for the Saito-Kurokawa lifting of Hilbert modular forms. We also want to introduce an example of a Hilbert-Siegel cusp form which satisfies the Maass relation.

11:00-12:00 **Hiro-aki Narita** (Kumamoto University)

Title: *Jacquet-Langlands-Shimizu correspondence for two theta lifts to* GSp(2) *and* GSp(1, 1)

Abstract: According to the Langlands principle of functoriality, an automorphic L-function of a reductive algebraic group G is expected to be some automorphic L-function of the quasi-split inner form G' of G. We take up the case of $G = GSp(1,1)$ and $G' = GSp(2)$. By theta lifts from $GO^*(4)$ we construct cusp forms on $GSp(1,1)_{\mathbb{A}_{\mathbb{Q}}}$ with the following properties:

(i) right-invariant by maximal open compact subgroups of $GSp(1,1)_{\mathbb{A}_f}$,

(ii) generating discrete series representations at ∞.

We show that the spinor L-functions of these cusp forms on $GSp(1,1)_{\mathbb{A}_{\mathbb{Q}}}$ coincide with those of paramodular cusp forms on $GSp(2)_{\mathbb{A}_{\mathbb{Q}}}$ (of some specified level) given by theta lifts from $GO(2,2)$. We also explicitly determine all local representations of irreducible cuspidal representations generated by the theta lifts to $GSp(1,1)_{\mathbb{A}_{\mathbb{Q}}}$ and $GSp(2)_{\mathbb{A}_{\mathbb{Q}}}$. We then see that the cuspidal representations generated by the two theta lifts satisfy the expected properties of "Jacquet-Langlands-Shimizu correspondence" locally and globally. (This is a joint work

with Takeo Okazaki and Ralf Schmidt.)

13:30-14:30 Masaaki Furusawa (Osaka City University)

Title: *On special values of certain L-functions*

Abstract: Let f be a holomorphic newform of weight k and let π denote the attached cuspidal representation of $\mathrm{G}L_2(\mathbb{A}_{\mathbb{Q}})$. Let V be a quadratic space defined over $\mathbb{Q}$ such that $V \otimes_{\mathbb{Q}} \mathbb{R}$ is anisotropic. Let τ be an irreducible automorphic representation of $\mathrm{S}O(V, \mathbb{A}_{\mathbb{Q}})$ whose infinity component τ_{∞} is the trivial representation. Then we prove an algebraicity result on the special value of $L_S(s, \pi \otimes \tau)$ at a point, which seems to be the rightmost critical point. As a special case we prove a new algebraicity result on the special value of the Rankin triple L-function for $\mathrm{G}L(2)$ in some unbalanced case, which conforms with Deligne's conjecture on special values of motivic L-functions made explicit by Blasius in the aforementioned case. This is a joint work with Kazuki Morimoto.

14:50-16:20 Hiroyuki Yoshida (Kyoto University)

Title: *Cohomology and L-values*

Abstract: In a paper published in 1959, Shimura presented an elegant calculation of the critical values of L-functions attached to elliptic modular forms using the first cohomology group. I will show that a similar calculation is possible for Hilbert modular forms over real quadratic fields using the second cohomology group. Explicit numerical examples calculated by this method will be presented. I will also show that we can deduce some information on periods which are not related to critical values by this method. A few open problems will also be discussed.

16:30- Orginizers

Title: *On the next RIMS conference*: Discussion and announcement

Jan.18 (Wed.)

9:45-10:45 Sho Takemori (Kyoto University)

Title: *p-adic Siegel-Eisenstein series of degree n*

Abstract: I define a Hilbert-Siegel Eisenstein series G of degree n, show an explicit formula of Fourier coefficients of G and state the relation between G and an Eisenstein series normally defined. Then, I mention that the Eisenstein series G is a p-adic limit of an Eisenstein series and G forms a p-adic family of Eisenstein series.

11:00-12:00 Yumiko Hironaka (Waseda University)

Title: *Spherical functions on the space of p-adic unitary hermitian forms*

Abstract: I will introduce the space of unitary hermitian matrices over a p-adic field with its Cartan decomoposition, and consider spherical functions on it. Then I will talk about their functional equations, explicit formulas, and parametrizations.

13:30-14:30 **Carel Faber** (KTH)

Title: *Modular forms and the cohomology of moduli space*

Abstract: Moduli spaces of curves and of abelian varieties possess cohomology classes corresponding to modular forms. Elliptic cusp forms give classes for curves of genus 1, and their generalizations, Siegel cusp forms, give classes for abelian varieties. One obtains a formula for moduli of pointed curves of genus 2 in these terms. For curves of higher genus, Teichmüller modular forms enter the picture. In the classical case, these were studied in detail by Ichikawa, but the forms here are in general vector valued. The related cohomology classes are still quite mysterious. (Joint work with Jonas Bergström and Gerard van der Geer.)

14:45-15:45 **William Duke** (UCLA)

Title: *Arithmetic properties of the Fourier coefficients of harmonic modular forms*

Abstract: In this talk I will report on joint work with Yingkun Li on some arithmetic properties of harmonic modular forms. These are Maass forms whose eigenvalue under the Laplacian is 0 and that are allowed to have polar-type singularities in the cusps. In recent years such forms of a half-integral weight $k < 1$, especially $k = 1/2$, have been the focus of a lot of research. In this work we consider the self-dual case of weight $k = 1$ where we observe that some connections with Galois representations exist for non-holomorphic harmonic forms.

16:00-16:50 **Minoru Hirose** (Kyoto University) , **Nobuo Sato** (Kyoto University)

Title: *On functional equation of the Shintani L-function*

Abstract: The Shintani L function was originally introduced by Takuro Shintani in his study on the Hecke L function. Like many zeta functions, the Shintani L function is defined by a certain sum. We extend the domains of its parameters using integral representation for the function, then prove its functional equation. The functional equation becomes quite simple if we rewrite it by the comlete Shinatni L function.

18:00- Banquet at *Camphora*

Jan.19 (Thurs.)

9:45-10:45 **Yuichi Hirano** (University of Tokyo)
Title: *Congruences of modular forms and the Iwasawa λ-invariants*

Abstract: We consider the problem to show how congruences between the Fourier coefficients of Hecke eigenforms of weight $k \geq 2$ and Eisenstein series of weight $k \geq 2$ give rise to corresponding congruences between the special values of the associated L-functions. In the case of weight $k = 2$, Vatsal obtained the congruences of L-functions. In this talk, we explain a recent result which generalizes his work to the case of higher weight $k \geq 2$. Then we apply it to the Iwasawa main conjecture for cusp forms of weight $k \geq 2$ in the special case

where p-adic Galois representations attached to cusp forms are residually reducible. This is a partial generalization of the results of Greenberg–Vatsal in the case of weight $k = 2$.

11:00-12:00 Cristian Virdol (Kyushu University)

Title: *Cyclicity of finite CM abelian varieties*

Abstract: I will compute the asymptotic formulas for the cyclicity of the orders of the reduction modulo $\wp$ of CM abelian varieties defined over number fields.

13:30-14:30 Lin Weng (Kyushu University)

Title: *Parabolic Reduction, Stability and Volumes of Fundamental Domains*

Abstract: In this talk, we expose an intrinsic structure of volumes of fundamental domains for reductive groups defined over number fields, in terms of parabolic reduction using stability. It claims that if v_P denotes the volume of the fundamental domain associated to the Levi of P and u_P denotes the volume of moduli space of semistable adelic points associated to the Levi of P, then $v_G = \sum_P d_P u_P$ and $u_G = \sum_P sgn(P) e_P v_P$ with d_P and e_P positive rational numbers, where P runs over all standard parabolics. This is obtained with the help of Arthur's analytic truncation, Lafforgue's arithmetic truncation, and Langlands' theory of Eisenstein series. A beautiful formula of Kontsevich for $SL(n, \mathbb{Z})$, one of our starting points, and some examples with lower ranks obtained by 足立憲治 will be given.

14:45-15:45 Lynne H. Walling (University of Bristol)

Title: *Hecke operators on degree 2 Eisenstein series and higher representation numbers*

Abstract: We discuss a technique for determining the action of Hecke operators on Siegel Eisenstein series of degree 2 and square-free level. We prove this space has a basis of eigenforms for the full Hecke algebra. The analysis leading to this also reveals relations among Eisenstein series provided the character is not primitive. We then realise average Siegel theta series of degree 2 on maximal lattices explicitly as linear combinations of Eisenstein series (the coefficients in these linear combinations are quite simple). Finally, we discuss how the above work leads to explicit closed-form formulas for average representation numbers of quadratic forms.

16:00-17:00 Tomoyoshi Ibukiyama(Osaka University), **Hidenori Katsurada** (Muroran Institute of Technology)

Title: *Exact critical values of the standard L-functions of vector valued Siegel modular forms*

Abstract: We will give exact critical values of standard zeta functions of certain vector values Siegel modular forms F of degree two. These are explicitly written by products of rational numbers, Petersson inner products (F, F) and powers of π. As an application, we give exact critical values of symmetric fourth L-function of the Ramanujan Delta function, which are conjectered by D. Zagier. The main technical points are differential operators and pullback formulas, and our works are also motivated by the Bloch-Kato conjecture. These points would be also shortly explained.

Jan.20 (Fri)

9:30-10:15 **Shingo Sugiyama** (Osaka University)
Title: *Reguralized periods of automorphic forms on* $\mathrm{G}L(2)$

Abstract: Let $\mathbb{A}$ be the adele ring of an algebraic number field. Tsuzuki introduced regularized periods to define periods of functions on $GL(2, \mathbb{A})$ which are not necessarily rapidly decreasing and gave the following results:

(1) Regularized periods of cusp forms which are associated with cuspidal automorphic representations of $GL(2)$ with square free conductor are explicitly described in terms of central L-values.

(2) Regularized periods of Eisenstein series constructed by induced representations from unramified characters are described in terms of Hecke L-functions.

We generalize these results (1) and (2) as follows:

(1$'$) Regularized periods of cusp forms which are associated with cuspidal automorphic representations of $GL(2)$ with arbitrary conductor are explicitly described in terms of central L-values.

(2$'$) Regularized periods of Eisenstein series constructed by induced representations from arbitrary characters are described in terms of Hecke L-functions.

In this talk we will give a definition of Tsuzuki's regularized periods and report computation of (1$'$) and (2$'$).

10:30-12:00 **Shunsuke Yamana** (Osaka City University)

Title: *Multiplicativity of the L-factors*

Abstract: One of the fundamental problems in the theory of automorphic representations is the definition of L and epsilon factors. To this problem, the doubling method of Piatetski-Shapiro and Rallis applies in the local situation to define the L-factor as a g.c.d. of the local zeta integrals for all good sections. In this talk I prove multiplicativity of this L factor. If time permits, we show that nonvanishing of global theta liftings is characterized in terms of the analytic properties of the complete L-functions and the occurrence in the local theta correspondence.