212. 個体領域ノ個數條件ニ就イテ.(II)

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Lemma 2. $F_m(a,A) \backsim (mx)A(x) \vee (\bar{A}(a) \,\&\, (m+1x)A(x))$

ココニ

$$F_m(a,A):\ \prod_{i=1}^{m}(x_i)\left\{\sum_{i,j}^{1.m}(x_i \neq x_j) \,\&\, \sum_{i=1}^{m}(x_i \neq a) \longrightarrow \prod_{i=1}^{m}A(x_i)\right\}$$

$$(mx)A(x):\ \prod_{i=1}^{m}(x_i)\left\{\sum_{i,j}^{1.m}(x_i \neq x_j) \longrightarrow \prod_{i=1}^{m}A(x_i)\right\}$$

証明:

先ヅ $F_m(a,A) \longrightarrow (mx)A(x) \vee \bar{A}(a)$ ....(i)

$\quad$ 〃 $\quad \longrightarrow (mx)A(x) \vee (m+1x)A(x)$ ....(ii)

ヲ証明スル。

(i)ヲ次ノ如ク変形スル。

$$F_m(a,A) \,\&\, A(a) \longrightarrow (mx)A(x)$$

然ルニ

$$A(a) \rightarrow \prod_{i=1}^{m}(x_i)\left\{\prod_{i,j}^{1.m}(x_i = x_j) \vee \sum_{i=1}^{m}(x_i \neq a) \vee \prod_{i=1}^{m}A(x_i)\right\} \quad (1)$$

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1) コノ証明ハ次ノ如クスレバヨイ。

$m=1$ ノトキハ $A(a) \backsim (x)(x \neq a \vee A(x))$ 成立スル.

m ノトキ成立スルト假定スル。

$$A(a) \backsim (x_{m+1})(x_{m+1} \neq a \vee A(x_{m+1}))$$

$$x_{m+1} \neq a \vee A(x_{m+1}) \to \prod_{i,0}^{1,m+1}(x_i = x_j) \vee x_{m+1} = a \vee \prod_{i=1}^{m+1} A(x_i)$$

故＝

$$A(a) \to \prod_{i=1}^{m+1}(x_i)\left\{\prod_{i,j}^{1,m+1}(x_i = x_j) \vee x_{m+1} = a \vee \prod_{i=1}^{m+1} A(x_i)\right\} \quad (1) \ .$$

又

$$\prod_{i,j}^{1,m}(x_i = x_j) \vee \sum(x_i \neq a) \vee \prod_{i=1}^{m} A(x_i)$$

$$\longrightarrow \prod_{i \neq j}^{1,m+1}(x_i = x_j) \vee \sum_{i=1}^{m}(x_i \neq a) \vee \prod_{i=1}^{m+1} A(x_i)$$

故＝

$$A(a) \to \prod_{i=1}^{m+1}(x_i)\left\{\prod_{i,j}^{1,m+1}(x_i = x_j) \vee \sum_{i=1}^{m}(x_i \neq a) \vee \prod_{i=1}^{m+1} A(x_i)\right\} \quad (2)$$

(1), (2) ヨリ

$$A(a) \to \prod_{i=1}^{m+1}(x_i)\left\{\prod_{i,j}^{1,m+1}(x_i = x_j) \vee \sum_{i=1}^{m+1}(x_i \neq a) \vee \prod_{i=1}^{m+1} A(x_i)\right\}$$

Q. E. D.

故＝ . $F_m(a, A)$ & $A(a)$

$$\to \prod_{i=1}^{m}(x_i)\left\{\prod_{i,j}^{1,m}(x_i = x_j) \vee \left[\prod_{i=1}^{m}(x_i = a) \right.\right.$$

$$\left.\left. \& \sum_{i=1}^{m}(x_i \neq a)\right] \vee \prod_{i=1}^{m} A(x_i)\right\},$$

右辺ハ $\prod_{i=1}^{m}(x_i)\left\{\prod_{i,j}^{1,m}(x_i=x_j)\vee\prod_{i=1}^{m}A(x_i)\right\}$ 即チ $(mx)A(x)$

故ニ $F_m(a,A)\ \&\ A(a)\longrightarrow(mx)A(x)$

(ii)ハ $(mx)A(x)\longrightarrow(m+1x)A(x)$ ヨリ

$$F_m(a,A)\longrightarrow(m+1x)A(x)$$

ヲ証明スレバヨイ。

Lemma 1 ヨリ

$$F_m(a,A)\longrightarrow A(x)\vee F_m(x,A)$$

α) ヨリ

$$F_m(a,A)\longrightarrow(x)\left\{A(x)\vee F_m(x,A)\right\}$$

$$\longrightarrow(m+1x)A(x)$$

次ニ逆ヲ証明スル。

$(mx)A(x)\longrightarrow F_m(a,A)$ ナルコトハ明カデアル故

$\bar{A}(a)\ \&\ (m+1x)A(x)\longrightarrow F_m(a,A)$ ヲ証明スル。

$$(m+1x)A(x)\longrightarrow A(a)\vee F_m(a,A)$$

$$\bar{A}(a)\ \&\ (m+1x)A(x)\longrightarrow\bar{A}(a)\vee\left(F_m(a,A)\ \&\ \bar{F}_m(a,A)\right)$$

$$\&\ A(a)\vee\bar{F}_m(a,A)$$

$$\longrightarrow\left(\bar{A}(a)\ \&\ A(a)\right)\vee F_m(a,A)\ \&\ \bar{A}(a)\vee\bar{F}_m(a,A)$$

$$\longrightarrow\left(\bar{A}(a)\ \&\ A(a)\right)\vee F_m(a,A)$$

$$\longrightarrow F_m(a,A)$$

故ニ $(mx)A(x)\vee\left\{\bar{A}(a)\ \&\ (m+1x)A(x)\right\}\longrightarrow F_m(a,A)$

Q. E. D.

$$F_m(a,A)\sim(mx)A(x)\vee\left\{\bar{A}(a)\ \&\ (m+1x)A(x)\right\}$$

ハ　$(x)(x \neq a \to A(x)) \backsim (x)A(x) \lor \{\bar{A}(a) \,\&\, (2x)A(x)\}$

ノ一ツ擴張デアル。之レニ就イテハ後ニ述ベタイト思フ。

次ニ、個数條件ニ就イテノ他ノ基本的ナ次ノ式ノ証明ヲ與ヘル。

$$(x)\Big(\sum_{i=1}^{m}(x \neq a_i) \longrightarrow A(x)\Big)$$

$$\backsim (x)A(x)$$

$$\lor \Big\{\prod_{i}^{1,m} H_1(a_i;A) \,\&\, ({}_2x)A(x)\Big\}$$

$$\text{- -}$$

$$\lor \Big\{\prod_{n_1,\cdots n_r}^{1,m} H_r(a_{n_1},a_{n_2},\cdots\cdots,a_{n_r};A) \,\&\, ({}_{r+1}x)A(x)\Big\}$$

$$\text{- -}$$

$$\lor \Big\{\prod H_m(a_1,a_2,\cdots\cdots,a_m;A) \,\&\, ({}_{m+1}x)A(x)\Big\}$$

ココニ,

$$H_1(a_i;A) : \quad \bar{A}(a_i)$$

$$H_m(a_1,a_2,\cdots\cdots,a_m;A) : \quad \sum_{i,j}^{1,m}(a_i \neq a_j) \,\&\, \sum_{i=1}^{m}\bar{A}(a_i).$$

一般ニ

$$H_r(a_{n_1},a_{n_2},\cdots\cdots,a_{n_r};A) : \quad \sum_{i,j}^{1,r}(a_{n_i} \neq a_{n_j}) \,\&\, \sum_{i=1}^{r}\bar{A}(a_i)$$

証明：　$m=1$ デ成立スル故，m デ成立スルモノト假定スル。上ノ式ニ於テ、$A(x)$ ノ代リニ $x=a_{m+1} \lor A(x) : \mathfrak{A}(x)$ ヲ代入スルトキハ．左辺ハ

$$(x)\Big(\sum_{i=1}^{m+1}(x \neq a_i) \longrightarrow A(x)\Big) \quad \text{トナル。}$$

右辺ハ $(x)\mathfrak{A}(x)$

$$\vee\left\{\prod_{i}^{1,m} H_1(a_i;\mathfrak{A})\,\&\,(2x)\mathfrak{A}(x)\right\}$$

- -

$$\vee\left\{\prod_{n_1,\cdots,n_r}^{1,m} H_r(a_{n_1},a_{n_2},\cdots\cdots,a_{n_r};\mathfrak{A})\,\&\,(r+1\,x)\mathfrak{A}(x)\right\}$$

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$$\vee\left\{\prod H_m(a_1,a_2,\cdots\cdots,a_m;\mathfrak{A})\,\&\,(m+1\,x)\mathfrak{A}(x)\right\}$$

$$(x)\mathfrak{A}(x):(x)(x\neq a_{m+1}\longrightarrow A(x))$$

$$\backsim (x)A(x)\vee\left\{\bar{A}(a_{m+1})\,\&\,(2x)A(x)\right\}$$

$$\longrightarrow (x)A(x)\vee\left\{\prod_{i}^{1,m+1} H_1(a_i;A)\,\&\,(2x)A(x)\right\}$$

一般ニ

$$\prod_{n_1,\cdots,n_r}^{1,m} H_r(a_{n_1},a_{n_2},\cdots\cdots,a_{n_r};\mathfrak{A})\,\&\,(r+1\,x)\mathfrak{A}(x)$$

$$\backsim \prod_{n_1,\cdots,n_r}^{1,m}\left\{H_r(a_{n_1},a_{n_2},\cdots\cdots,a_{n_r};A)\,\&\,\sum_{i=1}^{r}(a_{n_i}\neq a_{m+1})\right\}$$

$$\&\,F_{r+1}(a_{m+1};A)$$

Lemma 2ヨリ

$$F_{r+1}(a_{m+1},A)\sim(r+1\,x)A(x)\vee\left\{\bar{A}(a_{m+1})\,\&\,(r+2\,x)A(x)\right\}$$

故ニ上ノ式ハ

$$\left[\prod_{n_1,\cdots,n_r}^{1,m}\left\{H_r(a_{n_1},a_{n_2},\cdots\cdots,a_{n_r};A)\,\&\,\sum_{i=1}^{r}(a_{n_i}\neq a_{m+1})\right\}\right.$$

$$\left.\&\,(r+1\,x)A(x)\right]$$

$$\vee \left[\prod_{n_1,\cdots,n_r}^{1,m} H_{r+1}(a_{n_1}, a_{n_2}, \cdots\cdots, a_{n_r}, a_{m+1}; A) \,\&\, (r+2\,x) A(x) \right]$$

前項ニツイテハ

$$H_r(a_{n_1}, a_{n_2}, \cdots\cdots, a_{n_r}; A) \,\&\, \sum_{i=1}^{r} (a_{n_i} \neq a_{m+1})$$

$$\longrightarrow H_r(a_{n_1}, a_{n_2}, \cdots\cdots, a_{n_2}; A)$$

$$\longrightarrow \prod_{n_1,\cdots,n_r}^{1,m+1} H_r(a_{n_1}, \cdots\cdots, a_{n_r}; A)$$

$$\prod_{n_1,\cdots,n_r}^{1,m} \left\{ H_r(a_{n_1}, \cdots\cdots, a_{n_r}; A) \,\&\, \sum_{i=1}^{r} (a_{n_i} \neq a_{m+1}) \right\}$$

$$\longrightarrow \prod_{n_1,\cdots,n_r}^{1,m+1} H_r(a_{n_1}, a_{n_2}, \cdots\cdots, a_{n_r}; A)$$

後項ニツイテハ

$$\prod_{n_1,\cdots,n_r}^{1,m} H_{r+1}(a_{n_1}, a_{n_2}, \cdots\cdots, a_{n_r}, a_{m+1}; A)$$

$$\longrightarrow \prod_{n_1,\cdots,n_{r+1}}^{1,m+1} H_{r+1}(a_{n_1}, \cdots\cdots, a_{n_{r+1}}; A)$$

両者ヲ綜合シテ

$$\prod_{n_1,\cdots,n_r}^{1,m} H_r(a_{n_1}, a_{n_2}, \cdots\cdots, a_{n_r}, \mathfrak{A}) \,\&\, (r+1\,x)\, \mathfrak{A}(x)$$

$$\rightarrow \left[\prod_{n_1,\cdots,n_r}^{1,m+1} H_r(a_{n_1}, a_{n_2}, \cdots\cdots, a_{n_r}; A) \,\&\, (r+1\,x) A(x) \right]$$

$$\vee \left[\prod_{n_1,\cdots,n_{r+1}}^{1,m+1} H_{r+1}(a_{n_1}, a_{n_2}, \cdots\cdots, a_{n_{r+1}}; A) \,\&\, (r+2\,x) A(x) \right]$$

之ヨリ $$(x)\left(\sum_{i=1}^{m+1}(x \neq a_i) \longrightarrow A(x)\right)$$

$$\longrightarrow (x)A(x)$$

$$\vee\left\{\prod_{i}^{1,m+1} H_1(a_i;A) \& ({}_2x)A(x)\right\}$$

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$$\vee\left\{\prod_{n_1,\cdots,n_r}^{1,m+1} H_r(a_{n_1},\cdots\cdots,a_{n_r};A) \& ({}_{r+1}x)A(x)\right\}$$

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$$\vee\ \left\{\prod H_{m+1}(a_1,\cdots\cdots,a_{m+1};A) \& ({}_{m+2}x)A(x)\right\}$$

トナル、次ニコノ逆ヲ証明スル。

$$({}_{r+1}x)A(x) \longrightarrow (x)\left(\sum_{i=1}^{r}(x \neq a_{n_i}) \rightarrow A(x)\right)$$

$$\vee \bar{H}_r(a_{n_1},\cdots\cdots,a_{n_r};A)$$

$$(x)\left\{\sum_{i=1}^{r}(x \neq a_{n_i}) \longrightarrow A(x)\right\} \longrightarrow (x)\left\{\sum_{i=1}^{m+1}(x \neq a_i) \rightarrow A(x)\right\}$$

故ニ $$({}_{r+1}x)A(x) \longrightarrow (x)\left\{\sum_{i=1}^{m+1}(x \neq a_i) \longrightarrow A(x)\right\}$$

$$\vee \bar{H}_r(a_{n_1},\cdots\cdots,a_{n_r};A)$$

従ツテ

$$({}_{r+1}x)A(x) \longrightarrow (x)\left\{\sum_{i=1}^{m+1}(x \neq a_i) \longrightarrow A(x)\right\}$$

$$\vee \sum_{n_1,\cdots,n_r}^{1,m+1} \bar{H}_r(a_{n_1},\cdots\cdots,a_{n_r};A)$$

故ニ前ト同様ニシテ

$$\prod_{n_1,\cdots\cdots,n_r}^{1,m+1} H_r(a_{n_1},\cdots\cdots,a_{n_r};A)\,\&\,(r+1\,x)\,A(x)$$

$$\longrightarrow \prod_{n_1,\cdots\cdots,n_r}^{1,m+1} H_r(a_{n_1},\cdots\cdots,a_{n_r};A)\,\&\,(x)\left\{\sum_{i=1}^{m+1}(x \neq a_i) \longrightarrow A(x)\right\}$$

$$\vee \sum_{n_1,\cdots\cdots,n_r}^{1,m+1} \overline{H}_r(a_{n_1},\cdots\cdots,a_{n_r};A)$$

ヨリ $\longrightarrow (x)\left(\sum_{i=1}^{m+1}(x \neq a_i) \longrightarrow A(x)\right)$

上ノ式ハ各 Disjunktionsglied ヨリ出ル故、逆ガ又成立スル。

勿論コヽニ於ケル Induktion ハ前回同様有限的ノモノデアル。　——（未完）——