1120. 河口空間ノ共形幾何學

岩本　香行（東大學生）

§1. 次數 M ノ河口空間即チ曲線 $x^i(t)$ $(i=1,2,\cdots\cdots,n)$ ノ $t_0 \leqq t \leqq t_1$ ノ間ノ弧ノ長サガ積分

$$(1.1)\qquad S=\int_{t_0}^{t} F(x, x^{(1)}, \cdots\cdots, x^{(M)})\,dt$$

デ定義サレル空間ヲ考ヘル. コヽデ (11) ガ幾何學的意味ヲモツタメニハ, (11) ノ形ガ媒介變數 t ノスベテノ變換

$$(1.2)\qquad \bar{t}=\bar{t}(t)$$

ニ對シテ不變デナケレバナラナイ. (1.1) ニ對應シテ次數 M ノ線素 $(x^{(1)}, x^{(2)}, \cdots\cdots, x^{(M)})$ ノウケル變換

$$x^{i(\bar{\mu})}=\sum_{\nu=1}^{\mu} a_{\nu}^{\mu}\, x^{i(\nu)}$$

ハ M－徑數變換群ヲナシ, 無限小演子 $\Delta_1, \cdots\cdots, \Delta_M$ ハ

$$(1.3)\qquad \Delta_K=\sum_{\lambda \geqq K}\binom{\lambda}{K} x^{(\lambda-K+1)i}\frac{\partial}{\partial x^{(\lambda)i}}$$

デ與ヘラレル. Δ_K, Δ_H ノ Kommutator $[\Delta_K \Delta_H]$ ハ

$$(1.4)\quad [\Delta_K \Delta_H] = \frac{(K-H)(K+H-1)!}{K!\,H!}\Delta_{K+H-1}$$

トナル．之ヲ用ヒレバ，(1.1)ガ媒介変数ノ変換(1.2)デ不変デアルタメノ條件，即チ與ヘラレタスカラーFガ(1.2)デ

$$\overline{F}(x, x^{(\bar{1})}, \ldots\ldots, x^{(\bar{M})}) = \frac{dt}{d\bar{t}} F(x, x^{(1)}, \ldots\ldots, x^{(M)})$$

ノ如ク変換スルタメノ條件ハ次ノ形ニカクコトガ出来ル。

$$(1.5)\quad \Delta_1 F = F,\quad \Delta_K F = 0,\quad (2 \leqq K \leqq M)$$

$\rho(x, x^{(1)}, \ldots\ldots, x^{(\alpha)})$ハ座標$(x)$及ビ$\alpha$次マデノ線素ニdependスル任意ノ恒等的ニ零デナイスカラートスル。即チ

$$\Delta_K \rho = 0,\quad (1 \leqq K \leqq \alpha)$$

ヲ満足スル量トスルトキ，Fノ変化

$$(1.6)\quad \overline{F} = \rho F$$

ヲ河口空間Fノ次數αノ共形変換トイフ．コヽデ$0 \leqq \alpha \leqq M-1$トスル。

任意ノ点デ互ニα次ノ接觸ヲナス微小ナ曲線ノ弧ハ，α次ノ共形変換デstreckentreuニ寫像サレル。

(1.6)デ不変ナ量ヲ求メルタメニスカラーFニ関シテ次ノ假定ヲオク。

$\Delta_M F = 0$カラ$F_{(M)i}\, x^{(1)i} = 0$,

從ッテ$F_{(M)i\,(M)j}\, x^{(1)i} = 0$　從ッテ行列$(F_{(M)i(M)j})$

階数ハ高々 $n-1$ デアル。我々ハソレガ丁度 $n-1$ ニ等シイト假定スル。然ルトキハ $F_{(M)i\,(M)j}$ ニ関スル行列 $(F_{(M)i\,(M)j})$ ノ代数余因子ヲ H_1^{ij} トスレバ

$$H_1^{ij}=F_1\, x^{(1)i}x^{(1)j}$$

トナルヤウナ函数 F_1 ヲ求メルコトが出来ル。

F_1 ハ再ビ $F_{1\,(M)i\,(M)j}\, x^{(1)i}=0$ ナル関係ヲ満足スル。行列 $(F_{1\,(M)i\,(M)j})$ ノ階数が $n-1$ ニ等シイト假定シテ

$$\mathrm{Adj}\ F_{1\,(M)i\,(M)j}=H_2^{ij}$$

トオケバ $H_2^{ij}=F_2\, x^{(1)i}x^{(1)j}$ トナル様ナ F_2 が求マル。

F_1, F_2 ハ共形変換，媒介変数ノ変換及ビ座標変換デ次ノ如ク変換スル。

$$\overline{F}_1=\rho^{n-1}\left(\frac{d\bar{t}}{dt}\right)^{(2M-1)(n-1)+2}\left|\frac{\partial x}{\partial \bar{x}}\right|^2 F_1$$

$$\overline{F}_2=\rho^{(n-1)^2}\left(\frac{d\bar{t}}{dt}\right)^{((2M-1)n+3)(n-1)+2}\left|\frac{\partial x}{\partial \bar{x}}\right|^{2n} F_2$$

從ッテ $\overline{F_2^{-1}F_1^{n}}=\rho^{n-1}\left(\dfrac{dt}{d\bar{t}}\right)^{n-1}F_2^{-1}F_1^{n}$

故ニ $\theta=\mathrm{sgn}(F).\ F^{-1}\left|F_2^{-1}F_1^{n}\right|^{\frac{1}{n-1}}$

デ定義サレルθハ intrinsic ナスカラーデ且ツ共形不変量デアル。

$$\theta^{(1)}=\psi,\quad \kappa=F/\psi$$

トオケバ ψ ハ共形不変量デ媒介変数ノ変換デ F ト同ジ変換ヲウケ，κ ハ intrinsic デ共形変換デ

$$\bar{\kappa} = \rho \kappa$$

ノ如ク変換スル。

定理 2. (1.9) $$t_{i\mu} = \sum_{\lambda=\mu}^{M} \binom{\lambda}{\mu} \left(\frac{1}{F}\right)^{(\lambda-\mu)} E_{i\lambda},$$

$$(\mu = \alpha+1, \cdots\cdots, M)$$

ナル $M-\alpha$ コノ共変ベクトルハ，次数 α ノ共形変換ニ對シテ不変デアル。*

証明. $\bar{F} = \rho F$ カラ

$$\log \bar{F} = \log \rho + \log F$$

従ツテ $\mu \geqq \alpha+1$ ナラバ

$$\sum_{\lambda=\mu}^{M} (-1)^{\lambda} \binom{\lambda}{\mu} (\log \bar{F})_{(\lambda)i}^{\cdots(\lambda-\mu)} = \sum_{\lambda=\mu}^{M} (-1)^{\lambda} \binom{\lambda}{\mu} (\log F)_{(\lambda)i}$$

シカルニ

$$\sum_{\lambda=\mu}^{M} (-1)^{\lambda} \binom{\lambda}{\mu} (\log F)_{(\lambda)i}^{\cdots(\lambda-\mu)} = \sum_{\lambda=\mu}^{M} (-1)^{\lambda} \binom{\lambda}{\mu} \left(\frac{F_{(\lambda)i}}{F}\right)^{(\lambda-\mu)}$$

$$= \sum_{\lambda=\mu}^{M} \sum_{\nu=0}^{\lambda-\mu} (-1)^{\lambda} \binom{\lambda}{\mu} \binom{\lambda-\mu}{\nu} \left(\frac{1}{F}\right)^{(\nu)} F_{(\lambda)i}^{\cdots(\lambda-\mu-\nu)}$$

---

* $E_{i\lambda} = \sum_{\mu=\lambda}^{M} (-1)^{\mu} \binom{\mu}{\lambda} \left(F_{(\mu)i}\right)^{(\mu-\lambda)}$ ハ $0 \leqq \lambda \leqq M$ ニ對シ共変ベクトルノ成分ヲナシ Synge ノベクトルトヨバレル。

$$=\sum_{\lambda=\mu}^{M}\sum_{\nu=0}^{\lambda-\mu}-\binom{\mu+\nu}{\mu}\left(\frac{1}{F}\right)^{(\nu)}(-1)^{\lambda}\binom{\lambda}{\mu+\nu}F_{(\lambda)i}^{\,\cdots(\lambda-\mu-\nu)}$$

$$=\sum_{\alpha=\mu}^{M}\binom{\alpha}{\mu}\left(\frac{1}{F}\right)^{(\alpha-\mu)}\sum_{\lambda=\alpha}^{M}(-1)^{\lambda}\binom{\lambda}{\alpha}F_{(\lambda)i}^{\,\cdots(\lambda-\alpha)}$$

$$=\sum_{\lambda=\mu}^{M}\binom{\lambda}{\mu}\left(\frac{1}{F}\right)^{(\lambda-\mu)}E_{i\lambda}$$

即チ $t_{i\mu}=E_{i\mu}(\log F)$ $\bar{t}_{i\mu}=t_{i\mu}$ トナル。

$t_{i\alpha+1},\cdots\cdots,t_{iM}$ ハ媒介変数ノ変換デ不変デハナク

$$(1.10)\qquad \begin{aligned}&\Delta_K t_{i\mu}=-\binom{\mu+K}{K}t_{i\mu+K-1}\qquad 2\leqq K\leqq M-\mu+1\\&\Delta_1 t_{i\mu}=-\mu t_{i\mu}.\end{aligned}$$

ナル関係が成立スル．即チ

$$\Delta_K E_{i\mu}=-\binom{\mu+K-1}{K}E_{i\mu+K-1}-\delta_K^1 E_{i\mu}$$

及ビ

$$\Delta_K\left(\frac{1}{F}\right)^{(H)}=\frac{H+1-2K}{H+1}\binom{H+1}{K}\left(\frac{1}{F}\right)^{(H-K+1)}$$

ナル関係ヲ用フレバ

$$\Delta_K t_{i\mu}=\Delta_K\sum_{\nu=\mu}^{M}\binom{\nu}{\mu}\left(\frac{1}{F}\right)^{(\nu-\mu)}E_{i\nu}$$

$$=\sum_{\nu\geqq\mu}\left[\binom{\nu}{\mu}\frac{\nu-\mu-2K+1}{\nu\ \mu+1}\binom{\nu-\mu+1}{K}\left(\frac{1}{F}\right)^{(\nu-\mu-K+1)}E_{i\nu}\right.$$

$$\left.-\binom{\nu}{\mu}\left(\frac{1}{F}\right)^{(\nu-\mu)}\binom{\nu+K-1}{K}E_{i\nu+K-1}\right]$$

$$= \sum_{\nu \geq \mu+K-1} \left[\binom{\nu}{\mu} \frac{\nu-\mu-2K+1}{\nu-\mu+1} \binom{\nu-\mu+1}{K} - \binom{\nu-K+1}{\mu}\binom{\nu}{K} \right] \left(\frac{1}{A}\right)^{(\nu-\mu-K+1)} E_{i\nu}$$

$$= -\binom{\mu+K}{K} t_{i\mu+K-1}$$

$$\Delta_1 t_{i\mu} = -(\mu+1)\, t_{i\mu} + t_{i\mu} = -\mu\, t_{i\mu}$$

§2. 次ニρヲ定函数ト制限シタ場合ヲ論ジヨウ。コノ場合ニハMコノ*Synge*ノベクトルノミノ線状結合トシテ，共形不変デ且intrinsicナMコノ共変ベクトルヲ導クコトガ出来ル。

$E_{i\mu}$ハ共形変換デ次ノ如ク変換スル。

$$(2.1) \qquad \overline{E}_{i\mu} = c^{\lambda}_{\mu} E_{i\lambda}, \qquad c^{\lambda}_{\mu} = \binom{\lambda}{\mu} \rho^{(\lambda-\mu)}$$

之ハ(1.9)ヲ出シタト全ク同ジ計算デ証明スルコトガ出来ル。

$\overline{\kappa} = \rho\kappa$ カラ

$$\overline{\kappa}^{(\nu)} = \sum_{\mu=0}^{\nu} \binom{\nu}{\mu} \rho^{(\mu)} \kappa^{(\nu-\mu)}$$

従ッテ

$$(2.2) \qquad \frac{\rho^{(\nu)}}{\rho} = \frac{\overline{\kappa}^{(\nu)}}{\overline{\kappa}} - \frac{\kappa^{(\nu)}}{\kappa} - \sum_{\mu=1}^{\nu-1} \binom{\nu}{\mu} \frac{\rho^{(\mu)}}{\rho} \frac{F\kappa^{(\nu-\mu)}}{\kappa}$$

之ト $\qquad \overline{E}_{i1} = \sum_{0 \leq \nu \leq M-1} (\nu+1) E_{i\nu+1} \rho^{(\nu)}$

或ハ

$$\frac{\overline{E}_{i1}}{\overline{\varkappa}}=\sum_{\nu=0}^{M-1}(\nu+1)\frac{E_{i\nu+1}}{\varkappa}\frac{\rho^{(\nu)}}{\rho}$$

トカラ

$$\frac{\overline{E}_{i1}}{\overline{\varkappa}}=\sum_{\nu=0}^{M-2}{}'(\nu+1)\frac{E_{i\nu+1}}{\varkappa}\frac{\rho^{(\nu)}}{\rho}+\Bigg[\frac{\overline{\varkappa}^{(M-1)}}{\overline{\varkappa}}-\frac{\varkappa^{(M-1)}}{\varkappa}$$

$$-\sum_{\nu=1}^{M-2}\binom{M-1}{\nu}\frac{\rho^{(\nu)}}{\rho}\frac{\varkappa^{(M-\nu-1)}}{\varkappa}\Bigg]M\frac{E_{iM}}{\varkappa}$$

$$=\frac{E_{i1}}{\varkappa}+\sum_{\nu=1}^{M-2}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{i\nu+1}}{F\varkappa}-\binom{M-1}{\nu}\frac{\varkappa^{(M-\nu-1)}}{\varkappa}M\frac{E_{iM}}{\varkappa}\right]$$

$$+\left[\frac{\overline{\varkappa}^{(M-1)}}{\overline{\varkappa}}-\frac{\varkappa^{(M-1)}}{\varkappa}\right]M\frac{E_{iM}}{\varkappa}$$

従ッテ

$$(23)\quad \frac{\overline{E}_{i1}}{\overline{\varkappa}}-\frac{\overline{\varkappa}^{(M-1)}}{\overline{\varkappa}}M\overline{C}_{iM}=\frac{E_{i1}}{\varkappa}-\frac{\varkappa^{(M-1)}}{\varkappa}MC_{iM}$$

$$+\sum_{\nu=1}^{M-3}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{i\nu+1}}{\varkappa}-\binom{M-1}{\nu}\frac{\varkappa^{(M-\nu-1)}}{\varkappa}MC_{iM}\right]$$

$$+(M-1)C_{iM-1}\frac{\rho^{(M-2)}}{\rho}$$

$$C_{iM}=\frac{E_{iM}}{\varkappa},\quad C_{iM-1}=\frac{E_{iM-1}}{\varkappa}-\frac{\varkappa^{(1)}}{\varkappa}MC_{iM}$$

$E_{iM}=(-1)^M F_{(M)i}$，ヨリ直ニ $\overline{C}_{iM}=C_{iM}$

又 $\overline{C}_{iM-1}=C_{iM-1}$ デアル．之ヲ見ルニハ $F=\left(\frac{1}{\rho}\right)\overline{F}$，$\varkappa=\left(\frac{1}{\rho}\right)\overline{\varkappa}$

トシテ上ト同ジ計算ヲ繰返セバ

$$\frac{E_{i1}}{\varkappa}-\frac{\varkappa^{(M-1)}}{\varkappa}M\Gamma_{iM}=\frac{\bar{E}_{i1}}{\bar{\varkappa}}-\frac{\bar{\varkappa}^{(M-1)}}{\varkappa}M\bar{\Gamma}_{iM}$$

$$+\Phi\left(E_{iM},E_{iM-1},\cdots\cdots,\varkappa,\varkappa^{(1)},\cdots\cdots,\rho,\rho^{(1)},\cdots\rho^{(M-3)}\right)$$

$$-(M-1)\bar{\Gamma}_{iM-1}\frac{\rho^{(M-2)}}{\rho}$$

コノ式ト (2.3) トノ $\rho^{(M-2)}$ ヲ含ム項及ビ $\rho,\rho^{(1)}\cdots\cdots$ ヲ含マナイ項ヲ比較スレバ直ニ $\bar{\Gamma}_{iM-1}=\Gamma_{iM-1}$ ヲ得ル。之ヲ繰返シテ行ケバ

(2.4) $$\frac{\bar{E}_{i1}}{\bar{\varkappa}}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\bar{\varkappa}^{(M-\nu)}}{\bar{\varkappa}}\bar{\Gamma}_{iM-\nu+1}$$

$$=\frac{E_{i1}}{\varkappa}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\varkappa^{(M-\nu)}}{\varkappa}\Gamma_{iM-\nu+1}$$

$$-\sum_{\nu=0}^{M-\mu-1}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{i\nu+1}}{\varkappa}-\sum_{\alpha=0}^{\mu-1}\binom{M-\alpha-1}{\nu}\frac{\varkappa^{(M-\alpha-\nu-1)}}{\varkappa}(M-\alpha)\Gamma_{iM-\alpha}\right]$$

(2.5) $$\Gamma_{iM}=\frac{E_{iM}}{E\varkappa}$$

$$\Gamma_{iM-1}=\frac{E_{iM-1}}{F\varkappa}-\frac{\varkappa^{(1)}}{\varkappa}M\Gamma_{iM}$$

$$\Gamma_{iM-2}=\frac{E_{iM-2}}{\varkappa}-\frac{\varkappa^{(2)}}{\varkappa}\binom{M}{2}\Gamma_{iM}-\frac{\varkappa^{(1)}}{\varkappa}\binom{M-1}{1}\Gamma_{iM-1}$$

- -

$$\Gamma_{iM-\mu-1}=\frac{E_{iM-\mu-1}}{\varkappa}-\frac{\varkappa^{(\mu-1)}}{\varkappa}\binom{M}{\mu-1}\Gamma_{iM}$$

$$-\frac{x^{(\mu-2)}}{x}\binom{M-1}{\mu-2}\tau_{iM-1}-\cdots\cdots-\frac{x^{(1)}}{x}(M-\mu+2)\tau_{iM-\mu+2}$$

ヲ得ル．即チ（2.4）ヲ假定スレバ

$$\frac{\bar{E}_{i1}}{\bar{x}}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\bar{x}^{(M-\nu)}}{\bar{x}}\bar{\tau}_{iM-\nu+1}=\frac{E_{i1}}{x}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)x^{(M-\nu)}}{x}\tau_{iM-\nu+1}$$

$$+\sum_{\nu=0}^{M-\mu-2}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{i\nu+1}}{x}-\sum_{\alpha=0}^{\mu-1}\binom{M-\alpha-1}{\nu}\frac{x^{(M-\alpha-\nu-1)}}{x}(M-\alpha)\tau_{iM-\alpha}\right]$$

$$+\frac{\rho^{(M-\mu-1)}}{\rho}(M-\mu)\left[\frac{E_{iM-\mu}}{x}-\sum_{\alpha=0}^{\mu-1}\binom{M-\alpha}{M-\mu}\frac{x^{(\mu-\alpha)}}{x}\tau_{iM-\alpha}\right]$$

$$\tau_{iM-\mu}=\frac{E_{iM-\mu}}{x}-\sum_{\alpha=0}^{\mu-1}\binom{M-\alpha}{M-\mu}\frac{x^{(\mu-\alpha)}}{x}\tau_{iM-\alpha}$$

トオケバ之ハ共形不変デ

$$\frac{\bar{E}_{i1}}{\bar{x}}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\bar{x}^{(M-\nu)}}{\bar{x}}\bar{\tau}_{iM-\nu+1}=\frac{E_{i1}}{x}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)x^{(M-\nu)}}{x}\tau_{iM-\nu+1}$$

$$+\sum_{\nu=0}^{M-\mu-2}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{\lambda\nu+1}}{x}-\sum_{\alpha=0}^{\mu}\frac{x^{(M-\alpha-\nu-1)}}{x}(M-\alpha)\bar{\tau}_{iM-\alpha}\right]$$

$$+\left[\frac{\bar{x}^{(M-\mu-1)}}{\bar{x}}-\frac{x^{(M-\mu-1)}}{x}\right](M-\mu)\tau_{iM-\mu}$$

或ハ

$$\frac{\bar{E}_{i1}}{\bar{x}}-\sum_{\nu=1}^{\mu+1}\frac{(M-\nu+1)\bar{x}^{(M-\nu)}}{\bar{x}}\tau_{iM-\nu+1}=\frac{E_{i1}}{x}-\sum_{\nu=1}^{\mu+1}\frac{(M-\nu+1)x^{(M-\nu)}}{x}\tau_{iM-\nu+1}$$

$$+\sum_{\nu=0}^{M-\mu-2}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{i\nu+1}}{x}-\sum_{\alpha=0}^{\mu+1}\binom{M-\alpha-1}{\nu}\frac{x^{(M-\alpha-\nu-1)}}{x}(M-\alpha)\tau_{iM-\alpha}\right]$$

定理3. $$\mathfrak{C}_{iM-\nu}=\frac{E_{iM-\nu}}{\varkappa}-\sum_{\lambda=0}^{\nu-1}\binom{M-\lambda}{M-\nu}\frac{\varkappa^{(\nu-\lambda)}}{\varkappa}\mathfrak{C}_{iM-\lambda}$$

$$\mathfrak{C}_{iM}=\frac{E_{iM}}{\varkappa}$$

デ定義サレルMコノ共変ベクトル $\mathfrak{C}_{i1},\cdots\cdots,\mathfrak{C}_{iM}$ ハ共形不変量デアル。

(2.5) ヲ書キ直セバ

$$E_{iM}=\varkappa\,\mathfrak{C}_{iM}$$

$$E_{iM-1}=\varkappa^{(1)}M\,\mathfrak{C}_{iM}+\varkappa\,\mathfrak{C}_{iM-1}$$

$$E_{iM-2}=\varkappa^{(2)}\binom{M}{2}\mathfrak{C}_{iM}+\varkappa^{(1)}\binom{M-1}{1}\mathfrak{C}_{iM-1}+\varkappa\,\mathfrak{C}_{iM-2}$$

- - - - - - - - - - - - - - - - - -

$$E_{iM-\nu}=\varkappa^{(\nu)}\binom{M}{\nu}\mathfrak{C}_{iM}+\varkappa^{(\nu-1)}\binom{M-1}{\nu-1}\mathfrak{C}_{iM-1}$$

$$+\varkappa^{(\nu-2)}\binom{M-2}{\nu-2}\mathfrak{C}_{iM-2}+\cdots\cdots+\varkappa\,\mathfrak{C}_{iM-\nu}$$

従ッテ

$$\mathfrak{C}_{iM-\nu}=\frac{1}{\varkappa^{\nu+1}}\begin{vmatrix}\varkappa & & & & E_{iM}\\ M\varkappa^{(1)} & \varkappa & & & E_{iM-1}\\ \binom{M}{2}\varkappa^{(2)} & \binom{M-1}{1}\varkappa^{(1)} & \varkappa & & E_{iM-2}\\ \cdots & \cdots & \cdots & & \\ \binom{M}{\nu}\varkappa^{(\nu)} & \binom{M-1}{\nu-1}\varkappa^{(\nu-1)} & \binom{M-2}{\nu-2}\varkappa^{(\nu-2)} & \cdots & E_{iM-\nu}\end{vmatrix}$$

$$\mathfrak{C}_{iM-\nu}=\sum_{\mu=0}^{\nu}C_{M-\nu}^{M-\mu}E_{iM-\mu}$$

$$C^{M-\mu}_{M-\nu}=\frac{1}{\varkappa^{\nu+1}}\begin{vmatrix} \varkappa & & & & & 0 \\ M\varkappa^{(1)} & & & & & \vdots \\ \cdots\cdots & & & & & \vdots \\ \binom{M}{\mu}\varkappa^{(\mu)} & \binom{M-1}{\mu-1}\varkappa^{(\mu-1)} & \cdots & \varkappa & \cdots & 1 \\ \cdots\cdots & & & & & \vdots \\ \binom{M}{\nu}\varkappa^{(\nu)} & \binom{M-1}{\nu-1}\varkappa^{(\nu-1)} & & \cdots & \varkappa^{(1)} & 0 \end{vmatrix}$$

$$=\binom{M-\mu}{M-\nu}\left(\frac{1}{\varkappa}\right)^{(\nu-\mu)}$$

即チ

定理4. $\quad \mathfrak{C}_{i\mu}=\sum_{\lambda=\mu}^{M}C^{\lambda}_{\mu}E_{i\lambda},\quad C^{\lambda}_{\mu}=\binom{\lambda}{\mu}\left(\frac{1}{\varkappa}\right)^{(\lambda-\mu)}$

$E_{i\mu}$ ニ関シテハ基本的ナ関係

$$E_{i\mu}x^{(1)i}=0\quad(2\leqq\mu\leqq M),\quad E_{i1}x^{(1)i}=-F$$

ガ成立スル。之レカラ

$$\mathfrak{C}_{i\mu}x^{(1)i}=0,\quad(2\leqq\mu\leqq M),\quad \mathfrak{C}_{i1}x^{(1)i}=-\frac{F}{\varkappa}$$

定理5. $\quad \mathcal{E}_{i\lambda}=\frac{\varkappa}{F}\sum_{\mu=\lambda}^{M}A^{\mu}_{\lambda}\mathfrak{C}_{i\mu}$

トオケバ $\mathcal{E}_{i1}$ ハ intrinsic ナ共変ベクトルデ共形変換ニヨリ

$$\overline{\mathcal{E}}_{i\lambda}=\rho^{\mathcal{E}_{i\lambda}}$$

ノ如ク変換サレル．コヽニ $x^{(\mu)i}$，$\bar{t}=\bar{t}(t)$ ニ對スル変換ヲ

$$x^{(\bar{\mu})i} = \sum_{\lambda=1}^{\mu} a^{\mu}_{\lambda}\left(\frac{dt}{d\bar{t}}, \frac{d^2t}{d\bar{t}^2}, \cdots\cdots, \frac{d^{\mu-\lambda+1}t}{d\bar{t}^{\mu-\lambda+1}}\right) x^{(\lambda)i}$$

トシ A^{μ}_{λ} ハ a^{μ}_{λ} ノ $\frac{dt}{d\bar{t}}, \frac{d^2t}{d\bar{t}^2}, \cdots\cdots$ ノ所ヘ $\psi, \psi^{(1)} \cdots\cdots$ ヲ

代入シタモノ

即チ $\quad A^{\mu}_{\lambda} = a^{\mu}_{\lambda}(\psi, \psi^{(1)}, \cdots\cdots \psi^{(\mu-\lambda)})$

トスル。

証明. $E_{i\mu}$, C^{λ}_{μ}, A^{μ}_{λ} ハ $\bar{t} = \bar{t}(t)$ デ夫々次ノ如キ変換ヲウケルコトカラ明カデアル。*

$$\bar{E}_{i\mu} = \frac{dt}{d\bar{t}}\, \bar{a}^{\lambda}_{\mu} E_{i\lambda}$$

$$\bar{C}^{\lambda}_{\mu} = a^{\lambda}_{\alpha}\, \bar{a}^{\beta}_{\mu} C^{\alpha}_{\beta}$$

$$\bar{A}^{\mu}_{\lambda} = a^{\mu}_{\nu} A^{\nu}_{\lambda}$$

M−α コノ intrinsic ナ共変ベクトル

$$\varepsilon_{i\alpha+1}, \cdots\cdots, \varepsilon_{iM}$$

ハ次数 α ノ共形変換ニ對シ $\bar{\varepsilon}_{i\mu} = \rho\, \varepsilon_{i\mu}$ ノ如ク変換サレル。

$\varepsilon_{i\lambda}$ ハ又 $\xi_{i\lambda}$ ノ線状結合トシテ次ノ形ニカクコト

---

* A. Kawaguchi, Differentialgeometrie hohrer Ordnung. III

Jour Fac. Sci. Hokkaido Imp Univ (1941)

ガ出来ル。

$$\varepsilon_{i\lambda}=\sum_{\mu=\lambda}^{M}K_{\lambda}^{\mu}t_{i\mu}\qquad K_{\lambda}^{M}=\frac{\kappa}{\psi}\sum_{\alpha=\lambda}^{\mu}\binom{\mu}{\alpha}\psi^{(\mu-\alpha)}A_{\lambda}^{\alpha}(\psi,\psi^{(1)},\cdots,\psi^{(\alpha-\lambda)})$$

即チ $\dfrac{F}{\kappa}\varepsilon_{i\lambda}=\displaystyle\sum_{\lambda\leq\nu\leq M}\sum_{\nu\geq\mu}A_{\lambda}^{\mu}\binom{\nu}{\mu}\left(\frac{1}{n}\right)^{(\nu-\mu)}E_{i\nu}$

$$=\sum_{\lambda\leq\mu\leq M}\sum_{\mu\leq\nu\leq M}A_{\lambda}^{\mu}\binom{\nu}{\mu}\left(\frac{\psi}{F}\right)^{(\nu-\mu)}E_{i\nu}$$

$$=\sum_{\mu=\lambda}^{M}\sum_{\nu\geq\mu}^{M}\sum_{\alpha=0}^{\nu-\mu}\left(A_{\lambda}^{\mu}\binom{\nu}{\mu}\binom{\nu-\mu}{\alpha}\psi^{(\alpha)}\left(\frac{1}{F}\right)^{(\nu-\mu-\alpha)}E_{i\nu}\right.$$

$$=\sum_{\mu=\lambda}^{M}\sum_{\nu\geq\mu}^{M}\sum_{\alpha=0}^{\nu-\mu}A_{\lambda}^{\mu}\binom{\mu+\alpha}{\mu}\psi^{(\alpha)}\binom{\nu}{\mu+\alpha}\left(\frac{1}{F}\right)^{(\nu-\mu-\alpha)}E_{i\nu}$$

$$=\sum_{\alpha=\lambda}^{M}\left(\sum_{\mu=\lambda}^{\alpha}A_{\lambda}^{\mu}\binom{\alpha}{\mu}\psi^{(\alpha-\mu)}\right)\left(\frac{\nu}{\alpha}\right)\sum_{\nu=\alpha}^{M}\binom{\nu}{\alpha}\left(\frac{1}{F}\right)^{(\nu-\alpha)}E_{i\nu}$$

$$=\sum_{\alpha=\lambda}^{M}K_{\lambda}^{\alpha}t_{i\alpha}$$

§3. $g_{ij}-\psi^{2(M-1)}FF_{(M)i(M)j}+\varepsilon_{i1}\varepsilon_{j1}$

トオケバ g_{ij} ハ二階~~對称~~テンソルデソノ行列式ハ一般ニ0デナク,

$$g_{ij}x^{(1)i}=-F\varepsilon_{j1},\quad g_{ij}x^{(1)i}x^{(1)j}=F^2$$

ナル関係ガ成立シ, 且ツ共形変換デ

$$\bar{g}_{ij}=\rho^2 g_{ij}$$

ノ如ク変換スル。従ッテ g_{ij} ヲ我々ノ空間ノ基本テンソルトシテ採用スルコトガ出来ル。

X^j ヲ任意ノ *intrinsic* デ共形不変ナベクトルトスレバ

$$\psi^{2(M-1)}F.D_{jM-1}\left(F_{(M)i}\right)X^j=\psi^{2(M-1)}F.F_{(M)i(M)j}\frac{dX^j}{dt}$$

$$+\frac{1}{M}\psi^{2(M-1)}F\left[F_{(M)i(M-1)j}+\psi^{-1}\cdot\psi^{(1)}F_{(M)i(M)j}\right]X^j$$

$$\kappa\varepsilon_{i1}\left\{\left(\frac{\varepsilon_{j1}}{\kappa}X^j\right)^{(1)}\right\}=\varepsilon_{i1}\varepsilon_{j1}\frac{dX^j}{dt}+\kappa\varepsilon_{i1}\left[\left(\frac{\varepsilon_{j1}}{\kappa}\right)^{(1)}\right]X^j$$

ハ何レモ t ノ変換デ F ト同ジ変換ヲスル反変ベクトルデアル。之等カラ X^j ノ曲線ニ共形不変ナ共変微分

$$\frac{\delta X^j}{dt}=\frac{dX^j}{dt}+\Gamma^j_i X^i$$

$$\Gamma^j_i=\frac{1}{M}\psi^{-(M-1)}F\cdot g^{ja}\left[F_{(M)a(M-1)i}+\psi^{-1}\psi^{(1)}F_{(M)a(M)i}\right]$$

$$-\frac{\kappa}{F}x^{(1)j}\left(\frac{\varepsilon_{i1}}{\kappa}\right)^{(1)}$$

ヲ定義スルコトガ出来ル。

Γ^i_j ハ $2M+1$ 次ノ線素ニマデ *depend* スル函数デアル。

$$D_p\Gamma^i_j=\sum_{\lambda\geqq p}\binom{\lambda}{p}\Gamma^i_{j(\lambda)k}dx^{(\lambda-p)k}\qquad(p=1,2,\cdots\cdots)$$

カラ *intrinsic* デ共形不変ナ線型接続ノ係数

$$\sum_{0\leq\alpha\leq 2M} \Gamma^{i(\alpha)}_{jk}\, dx^{(\alpha)k}$$

ヲ導クコトガ出来ル. 之ヲ用ヒテ X^i ノ共変微分

$$\delta X^i = dX^i + \sum_{a=0}^{2M+1} \Gamma^{i(a)}_{jk}\, X^j dx^{(a)k}$$

ガ定義サレル。

共形不変ナ基本接続ノ決点, ρ ガ一般ノ函数ノ場合ノ理論, 或ハ共変微分ノ次数ヲモ少シ低クスルコトハ出来ナイカ, 一点 (x) ニ於テ相隣ルニツノ線素 $(x^{(1)}, \dots\dots, x^{(\nu)})$, $(x^{(1)}+dx^{(1)}, \dots\dots, x^{(\nu)}+dx^{(\nu)})$ ノ間ノ角ヲ定義スルニハドウスレバヨイカ等ノ問題ガ残ッテヰルガ今回ハ之レデ筆ヲオク。