Abstract

Course Lecture 1

Extremal Kähler 計量の存在問題について

(The existence problem of extremal Kähler metrics on polarized algebraic manifolds)

Toshiki MABUCHI (満渕　俊樹) (Osaka U.)

アブストラクト: Kähler-Einstein 計量の存在問題に関する Yau-Tian-Donaldson 予想は最近 Tian, Chen-Donaldson-Sun によって肯定的に解決しました．しかしながら一般偏極の場合や，より一般的には extremal Kähler 計量の存在問題については殆ど何も知られていません．この mini-lectures では，extremal Kähler 計量の存在問題を論じるつもりですが，Kähler-Einstein 計量の場合と extremal Kähler 計量の違いを強調して議論を進めていきたいと思っています．

Abstract: The Yau-Tian-Donalkdson Conjecture on the existence of Kähler-Einstein metrics is recently solved affirmatively by Tian and Chen-Donaldson-Sun. In a series of talks, I'll discuss the existence problem of extremal Kähler metrics with emphasis on the differences between Kähler-Einstein metrics and extremal Kähler metrics.

Course Lecture 2

小林双曲性と高次元 **Nevanlinna** 理論

(Kobayashi hyperbolicity and higher dimensional Nevanlinna theory)

Katsutoshi YAMANOI(山ノ井克俊） (Titech)

アブストラクト： 1960 年代に小林昭七先生は、小林双曲多様体の概念を導入し、その様々な基本的性質を証明することで現在の小林双曲多様体論の礎を確立した。

小林双曲多様体論の重要な問題の一つは、与えられた複素射影多様体 X が小林双曲的か否かを判定することである。そのためには、X の中に非定値正則曲線 $\mathbb{C} \to X$ が存在するかどうかを調べればよいことが知られている（Brody 判定法)。正則曲線 $\mathbb{C} \to X$ の挙動を調べるための重要な方法の一つが、正則曲線 $\mathbb{C} \to X$ と X の因子あるいは直線束との交差理論である高次元 Nevanlinna 理論である。特に、高次元 Nevanlinna 理論における基本予想である第二主要予想が重要な役割を果たす。この講演では、小林双曲多様体と高次元 Nevanlinna 理論について、予備知識を仮定せずに背景も含めつつお話ししたい。

Abstract: In 1960s, Professor Shoshichi Kobayashi established the theory of hyperbolic complex manifolds, which is now called Kobayashi hyperbolic manifolds.

One interesting problem in the theory is to find or characterize Kobayashi hyperbolic manifolds in complex projective manifolds. It is known, due to Brody, that complex projective manifolds X are Kobayashi hyperbolic if and only if there are no non-constant holomorphic maps $\mathbb{C} \to X$. An important tool to deal with holomorphic maps $\mathbb{C} \to X$ is higher dimensional Nevanlinna theory, which can be considered as intersection theory between holomorphic maps $\mathbb{C} \to X$ and divisors or line bundles on X. Especially, the second fundamental conjecture in higher dimensional Nevanlinna theory is important. In this talk, we shall discuss about Kobayashi hyperbolic manifolds and higher dimensional Nevanlinna theory, assuming no preliminary knowledge about the theories.

Invited Talks

Twistor spaces of Spin(7)-manifolds

Ichiro ENOKI (Osaka U.)

Abstract: A general construction of twistor spaces as almost complex manifold for an arbitrary G-manifold with a connection has been proposed by Berard-Bergery and Ochiai. They described the integrabity condition of the almost complex structure in terms of the curvature and the torsion of this connection. In this talk we will show that each holonomy Spin(7)-manifold has 2 kinds of twistor spaces whose almost complex structures are integrable.

On the book "*Transformation groups in Differential Geometry*"

Akito FUTAKI (U. Tokyo)

Abstract: I will talk on my research inspired by Professor Kobayashi's book "Transformation groups in Differential Geometry". In fact in my research I repeatedly used a single idea for which I got a hint from this book.

Dupin hypersurfaces in Lie sphere geometry

Gary JENSEN (Washington U.)

Abstract: Why has Lie sphere geometry been an effective tool in the classification of Dupin hypersurfaces in spheres? By the method of moving frames, the classification of isoparametric surfaces in the classical geometries is an elementary exercise. The same proof classifies the cyclides of Dupin in Möbius geometry and Lie sphere geometry. Much of this proof extends to the cases of higher dimensions and greater number of principal curvatures.

Brody curves and mean dimension

Shinichiro MATSUO (Osaka U.)

Abstract: This talk is based on a joint work with Masaki Tsukamoto of Kyoto University.

A BRODY CURVE is studied in Nevanlinna theory. It is an entire holomolphic map with its sup norm of the derivative uniformly bounded. The moduli space of all Brody curves turns out to be infinite dimensional.

Gromov introduced MEAN DIMENSION as "dimension of such infinite dimensional spaces".

We have studied mean dimension of the moduli spaces of all Brody curves. In particular, we give the exact formula of the mean dimension of the moduli space of Brody curves to the Riemann sphere.

On Kobayashi hyperbolicity of Zariski-generic projective algebraic hypersurfaces

Joël MERKER (U. Paris-Sud)

Abstract: Professor Shoshichi Kobayashi's complete works include aspects covering exterior differential systems and complex algebraic geometry as well.

This talk aims at presenting a few programmatic approaches towards effective Čech cohomology that would construct explicit sections of certain high rank (vector) jet bundles by developing new tools in algebraic geometry that are deeply related to Cartan's theory of curved geometries, the modernization of which was mainly achieved in the monograph of Kobayashi and Nomizu.

Since 1970, Kobayashi's conjecture that Zariski-generic projective algebraic hypersurfaces $X^n \subset \mathbb{P}^{n+1}(\mathbb{C})$ of sufficiently high degree should be Kobayashi- (or equivalently Brody-) hyperbolic is not yet completely settled, while the very first examples in arbitrary dimension were produced by Masuda and Noguchi in 1996.

A report on the current state of the art will exhibit the partly hidden fantastic computational complexity of the yet unstudied, labyrinthic, cohomologies.

Hypersurface geometry and expression via the moment map

Reiko MIYAOKA (Tohoku U.)

Abstract: There is a celebrated paper "Minimal embeddings of R-spaces" by S. Kobayashi and M. Takeuchi (JDG,1968). Isoparametric submanifolds in spheres of codimension greater than one are known to be R-spaces, and so all homogeneous (1991). On the other hand, hypersurface case is complicated by the presence of infinitely many non-homogeneous examples linked to the representation of Clifford algebras. Recently, the classification made remarkable progress in this case. We report it, and also give an expression of hypersurfaces via the moment map of the spin action.

Refined Chern-Simons theory and Hilbert schemes of points on the plane

Hiraku NAKAJIMA (RIMS)

Abstract: Aganagic and Shakirov propose a refinement of the $SU(N)$ Chern-Simons theory for links in three manifolds with S^1-symmetry, such as torus knots in S^3, based on deformation of the S and T matrices, originally found by Kirillov and Cherednik. We relate the large N limit of the S matrix to the Hilbert schemes of points on the affine plane. As an application, we find an explicit formula for the Euler characteristics of the universal sheaf, applied arbitrary Schur functor.

Kobayashi hyerbolicity and Lang's conjecture

Junjiro NOGUCHI (U. Tokyo)

Abstract: S. Kobayashi firstly announced a canonical way of constructing a new pseudo-distance d_M on each complex manifold M (Bull. A.M.S. 73 (1967)). He then discussed its properties in more details in his paper (J. Math. Soc. Jpn. 19 (1967)) and a monograph (Marcel Dekker, New York, 1970). Then the notion of the Kobayashi hyperbolicity attracted attention and interest from a number of branches of mathematics. S. Lang interested in it from the view point of Diophantine approximation theory, in particular of the problem of rational points (Bull. A.M.S. 80 (1974)). He raised a number of interesting problems, conjectures and the analogues over function fields, as well. The problem has naturally a connection to the Nevanlinna theory for entire curves in algebraic varieties and Vojta's Conjecture. In this talk I will discuss the developments of these problems, some results and obtained examples.

Levi flats in Hopf surfaces

Takeo OHSAWA (Nagoya U.)

Abstract: A compact Levi flat hypersurface in a complex manifold is said to be of q-concave type if it admits a neighborhood system consisting of q-concave manifolds in the sense of Andreotti- Grauert. The real analytic Levi flat hypersurfaces of 1-concave type in Hopf surfaces are classified.

Geometry of symmetric R-spaces

Makiko SUMI TANAKA (TUS, Noda)

Abstract: A symmetric R-space is a compact Riemannian symmetric space which is realized as a linear isotropy orbit of certain Riemannian symmetric space of compact type. Since every Hermitian symmetric space of compact type is realized as an adjoint orbit of a compact semisimple Lie group, it is a symmetric R-space. In 1965 T. Nagano introduced the notion of symmetric R-space as a compact Riemannian symmetric space endowed with transitive action of noncompact Lie group which contains the isometry group of M. Then, S. Kobayashi constructed explicitly the embedding of some symmetric R-spaces into the Euclidean space. In his joint paper with M. Takeuchi in 1968, they proved that every R-space, which is a kind of generalization of a symmetric R-space, has a natural embedding into the Euclidean space which is a minimum embedding. Since then, a lot of researches on symmetric R-spaces as submanifolds in the Euclidean space have been done up to now. In my recent joint work with H. Tasaki we investigated fundamental properties of antipodal sets in a symmetric R-space by making use of the natural embedding. An antipodal set in a compact symmetric space is a finite subset in which the geodesic symmetry at each point is the identity. We also proved that the intersection of two real forms in a Hermitian symmetric space of compact type is

an antipodal set, where it is known that every real form in a Hermitian symmetric space of compact type is a symmetric R-space, and vice versa.

Constructions of pseudoeffective metrics on relative adjoint line bundles

Hajime TSUJI (Sophia U.)

Abstract: We construct singular hermitian metric on the relative adjoint line bundle on a smooth Kähler family under the natural condition. by two different methods:

(1) Using the invariant volume form defined by the extremal property,

(2) Using the Kähler-Ricci flow.

Lang exceptional set for integral points

Paul VOJTA (UC, Berkeley)

Abstract: In 1986 and 1991, S. Lang defined holomorphic, diophantine, and geometric exceptional sets of a complete variety over $\mathbb{C}$, over a number field, or over a field of characteristic zero, respectively, and conjectured that they should coincide when defined. This talk will consider the possibility of extending this definition to holomorphic curves or integral points in quasi-projective varieties. A central question that arises is, given an abelian (or semiabelian) variety A and a Zariski-closed subset Z of codimension ≥ 2, can one find a nonconstant holomorphic curve in $A \setminus Z$ with Zariski-dense image, or a Zariski-dense set of integral points on $A \setminus Z$? For holomorphic curves this is quite easy. This talk will discuss the question for integral points.

Metric methods in algebraic geometry : in memory of Professor Kobayashi

Shing-Tung YAU (Harvard U.)

Abstract: In this talk, I shall discuss metric invariants associated to complex manifolds. Some of the ideas dates back to Prof Kobayashi. Some of them were developed by me and my students. An important one is the one appeared in the thesis of Chi. Some method of calculations will be discussed.