

第42回変換群論シンポジウム アブストラクト

11月26日

Definable C^r submanifolds in a definable C^r manifold

川上智博 (和歌山大学)

Abstract. Let X be a definable C^r manifold, Y_1, Y_2 definably compact definable C^r submanifolds of X such that $\dim Y_1 + \dim Y_2 < \dim X$ and Y_1 has a trivial normal bundle. We prove that there exists a definable isotopy $\{h_p\}_{p \in J}$ such that $h_0 = id_X$ and $h_1(Y_1) \cap Y_2 = \emptyset$.

Equivariant maps between representation spheres of cyclic p -groups

大橋耕 (東京大学)

Abstract. We will consider necessary conditions for the existence of equivariant maps between the unit spheres of unitary representations of a cyclic p -group G . Bartsch gave a necessary condition for some unitary representations of G by using equivariant K -theory. In this talk, we will give two necessary conditions following Bartsch's approach. One is a generalization of Bartsch's result for all unitary representations of G with fixed point free actions. The other gives a stronger necessary condition under stronger assumptions.

Hom 複体を与えるグラフの彩色数の下界について

松下尚弘 (東京大学)

Abstract. 単純グラフ G に対し、辺で結ばれている頂点では異なるように、 G の頂点に色を与えることを G の彩色という。 G の彩色に必要な色の個数を G の彩色数といい、 $\chi(G)$ で表す。

Hom 複体とは、二つのグラフ T, G に対して定義される CW 複体であり、 $\text{Hom}(T, G)$ で表す。任意のグラフ G に対し、

$$\chi(G) > \text{conn}(\text{Hom}(T, G)) + \chi(T)$$

なる不等式が成り立つとき、 T をホモトピーテストグラフであるという。ここで、 $\text{conn}(X)$ は、位相空間 X が n -連結となる最大の (-1) 以上の整数 (ただし $X = \emptyset$ のときは $-\infty$ とする) である。ホモトピーテストグラフの例としては、 $n \geq 2$ に対する完全グラフ K_n (Lovász, Babson-Kozlov) や、奇数次のサイクル C_{2r+1} (Babson-Kozlov) などが知られている。

しかし $T = K_2$ のときは、Hom 複体の与える彩色数の下限と、実際の彩色数とが一致しない例が知られている。特に Walker は 1983 年の論文において、「任意の正の整数 n に対し、上記の下界と、 G の彩色数が n 以上差がある G の例」や「 $\text{Hom}(K_2, G)$ -複体がホモトピー同値だが、彩色数が 1 異なる例」を発見している。

本講演では、上の Walker の結果を、以下のように一般化することを考える。任意の有限グラフ T と、彩色数が 3 以上のグラフ G 、および任意の整数 n に対して、 G を部分グラフとして含む H であって、以下の二つの性質を満たすものが存在する。一つの性質は包含 $\text{Hom}(T, G) \rightarrow \text{Hom}(T, H)$ がホモトピー同値であること、もう一つは H の彩色数が n より大きいことである。特に任意の有限グラフ T に対して、 $\text{Hom}(T, G)$ のホモトピー不変量は G の彩色数の上界を与えないことがこのことからわかる。

11月27日

The intersection of two real flag manifolds in a complex flag manifold
酒井高司 (首都大学東京)

Abstract. Tanaka and Tasaki studied the antipodal structure of the intersection of two real forms in Hermitian symmetric spaces of compact type. An orbit of the adjoint representation of a compact connected Lie group G admits a G -invariant Kähler structure, and called a complex flag manifold. Furthermore, any simply-connected compact homogeneous Kähler manifold is a complex flag manifold. Using a torus action, we can define (generalized) antipodal sets of a complex flag manifold. An orbit of the linear isotropy representation of the compact symmetric space G/K is called a real flag manifold, and is embedded in a complex flag manifold as a real form. In this talk, we will give a necessary and sufficient condition for two real flag manifolds, which are not necessarily congruent with each other, in a complex flag manifold to intersect transversally in terms of symmetric triads. Moreover we will show that the intersection is an orbit of a certain Weyl group and an antipodal set, if the intersection is discrete. This talk is based on a joint work with Osamu Ikawa, Hiroshi Iriyeh, Takayuki Okuda and Hiroyuki Tasaki.

**Totally geodesic surfaces in Riemannian symmetric spaces and
nilpotent orbits**
奥田隆幸 (広島大学)

Abstract. Let $X = G/K$ be a Riemannian symmetric space of non-compact type with connected G . We are interested in classifications of totally geodesic (complete) submanifolds in $X = G/K$. It is known that any totally geodesic submanifold of X is homogeneous, more precisely, any such submanifold can be realized as an orbit of a reductive subgroup L of G acting on X . In this talk, we give a one-to-one correspondence between the set of G -conjugate classes of non-flat totally geodesic oriented surfaces in the Riemannian symmetric space $X = G/K$ and the set of nilpotent orbits of the real semisimple Lie algebra $\text{Lie } G$, which were already classified in Representation theory. This is joint work with Akira Kubo (Hiroshima Shudo Univ.), Katsuya Mashimo (Hosei Univ.) and Hiroshi Tamaru (Hiroshima Univ.).

Combination of Lorentzian transformation groups

増田高行 (大阪大学)

Abstract. We will consider classification of affine Lorentzian transformation groups acting on $(2 + 1)$ -Minkowski spacetime properly discontinuously. All noncocompact affine Lorentzian transformations acting properly discontinuously are obtained by affine deformations of noncompact hyperbolic surfaces. Let a hyperbolic surface fixed. We introduce a new parameter, the affine twist parameter. Then we show that the affine deformation space can be parametrized by Margulis invariants and affine twist parameters. We will also talk about some topics associated with this theory.

A normal generating set for the Torelli group of a compact non-orientable surface

小林竜馬 (石川工業高等専門学校)

Abstract. The mapping class group $M(N)$ of a compact non-orientable surface N is defined as the group consisting of isotopy classes of diffeomorphisms over N fixing the boundary pointwise. The Torelli group $I(N)$ of a compact non-orientable surface N is defined as the subgroup of $M(N)$ consisting of mapping classes acting trivially on the integral first homology group of N . Hirose and the speaker have obtained a normal generating set for $I(N)$, where N is a genus $g(> 3)$ closed non-orientable surface. In this work, we obtain a normal generating set for $I(N)$, where N is a genus $g(> 3)$ compact non-orientable surface with $b(> 0)$ boundary components.

Cohomology of non-orientable toric origami manifolds

曾昊智 (大阪市立大学)

Abstract. Toric origami manifolds, introduced by A. Canas da Silva, V. Guillemin and A. R. Pires, are generalization of symplectic toric manifolds. In this talk we will discuss cohomology groups of some kinds of toric origami manifolds. This talk is based on the joint work with Anton Ayzenberg, Mikiya Masuda and Seonjeong Park.

有理点へのホモトピー論的手法
南範彦 (名古屋工業大学)

Abstract. 近年ホモトピー論が有理点研究に応用されている。本講演ではこの状況をホモトピー論的観点から概観する。可能であれば、あるホモトピー論手法の有理点への応用の可能性についても、言及したい。

11月28日

A_5 の Burnside 環について
杉村匡郁 (岡山大学)

Abstract. Let G be a finite nontrivial group and \mathcal{F} a set of subgroups of G which is closed under conjugations by elements in G and under taking subgroups. Let \mathfrak{F} denote the category whose objects are elements in \mathcal{F} and whose morphisms are triples (H, g, K) such that $H, K \in \mathcal{F}$ and $g \in G$ with $gHg^{-1} \subset K$. We denote by $A(G)$ the Burnside ring of G . For each morphism (H, g, K) , we have the associated homomorphism $(H, g, K)^* : A(K) \rightarrow A(H)$. In particular, if $H \leq K$ then $(H, e, K)^*$ agrees with $\text{res}_H^K : A(K) \rightarrow A(H)$. We denote by $A(\mathfrak{F})$ the inverse limit

$$\text{inv-lim}_{\mathfrak{F}} A(\bullet) \left(\subset \prod_{H \in \mathcal{F}} A(H) \right)$$

associated with the category \mathfrak{F} . We denote by $\text{res}_{\mathcal{F}}$ the restriction homomorphism $A(G) \rightarrow A(\mathfrak{F})$ and by $A(G)|_{\mathcal{F}}$ the image of the map $\text{res}_{\mathcal{F}}$. It is interesting to ask whether $A(G)|_{\mathcal{F}}$ coincides with $A(\mathfrak{F})$. Y.Hara and M.Morimoto showed that in the case of $G = A_4$, alternating group on four letters, the answer is affirmative. We consider that issue in the case of $G = A_5$.

閉多様体の A_5 -不動点集合としての実現

森本雅治（岡山大学）

Abstract. Let G be the alternating group on 5 letters and let \mathfrak{M} denote the family of closed smooth manifolds which can be obtained as the G -fixed point sets of smooth G -actions on disks. Let S^n denote the sphere of dimension n , let $P_{\mathbb{C}}^n$ and $P_{\mathbb{R}}^n$ denote the complex and real projective space of dimension n , respectively, and let L_m^{2n-1} denote the lens space $S(\mathbb{C}^n)/C_m$, where m is an integer ≥ 3 and

$$C_m = \{z \in \mathbb{C} \mid z^m = 1\}.$$

Let $M \in \mathfrak{M}$. We will discuss whether M can be realized as the G -fixed point sets of smooth G -actions on S^n , $P_{\mathbb{C}}^n$, $P_{\mathbb{R}}^n$ and L_m^{2n-1} .

String topology of the Borel constructions

内藤貴仁（東京大学）

Abstract. The theory of string topology is a study of algebraic structures on the homology of the free loop space (called the loop homology). It is known that the loop homology of a manifold has many algebraic structures, for example graded commutative algebra, Batalin-Vilkovisky algebra and 2-dimensional TQFT. In this talk, we will study string topology of the Borel constructions, especially TQFT structure on the loop homology. Moreover, we will introduce some properties and give some computational examples of the loop homology.

Gysin formulas for the universal Hall-Littlewood functions

中川征樹 (岡山大学)

Abstract. For certain kinds of maps (e.g., smooth maps between compact, oriented manifolds, or projections of fiber bundles), the *Gysin maps* (sometimes called *push-forwards*, *Umkehr maps*, *integration over the fiber* etc.) can be defined in ordinary cohomology. In *Schubert calculus*, there are many formulas for Gysin maps for Grassmann and flag bundles which relate *Schubert classes* with *Schur S - and P -functions* (Damon, Fulton, Harris-Tu, Pragacz). Recently Pragacz generalized the above formulas to the *Hall-Littlewood functions* which interpolate Schur S -functions and P -functions. Our main goal is to generalize the above formulas in ordinary cohomology to the *complex cobordism theory* which is *universal* among *complex-oriented generalized cohomology theories*. More precisely, we introduce the *universal* analogue of the Hall-Littlewood functions, which we call the *universal Hall-Littlewood functions*, and give analogous Gysin formulas in complex cobordism theory. This is joint work with H. Naruse.