Remarks on the Burnside ring of A_5

Masafumi Sugimura

Abstract

Let G be a finite nontrivial group and $\mathcal{F}$ a set of subgroups of G which is closed under conjugations by elements in G and under taking subgroups. Let $\mathfrak{F}$ denote the category whose objects are elements in $\mathcal{F}$ and whose morphisms are triples (H, g, K) such that H, $K \in \mathcal{F}$ and $g \in G$ with $gHg^{-1} \subset K$. We denote by $A(G)$ the Burnside ring of G. For each morphism (H, g, K), we have the associated homomorphism $(H, g, K)^* : A(K) \to A(H)$. In particular, if $H \leq K$ then $(H, e, K)^*$ agrees with $\mathrm{res}^K_H : A(K) \to A(H)$. We denote by $A(\mathfrak{F})$ the inverse limit

$$\text{inv-lim}_{\mathfrak{F}} A(\,\bullet\,) \quad \left(\subset \prod_{H \in \mathcal{F}} A(H) \right)$$

associated with the category $\mathfrak{F}$. We denote by $\mathrm{res}_{\mathcal{F}}$ the restriction homomorphism $A(G) \to A(\mathfrak{F})$ and by $A(G)|_{\mathcal{F}}$ the image of the map $\mathrm{res}_{\mathcal{F}}$. It is interesting to ask whether $A(G)|_{\mathcal{F}}$ coinsides with $A(\mathfrak{F})$. Y.Hara and M.Morimoto showed that in the case of $G = A_4$, alternating group on four letters, the answer is affirmative. We consider that issue in the case of $G = A_5$.