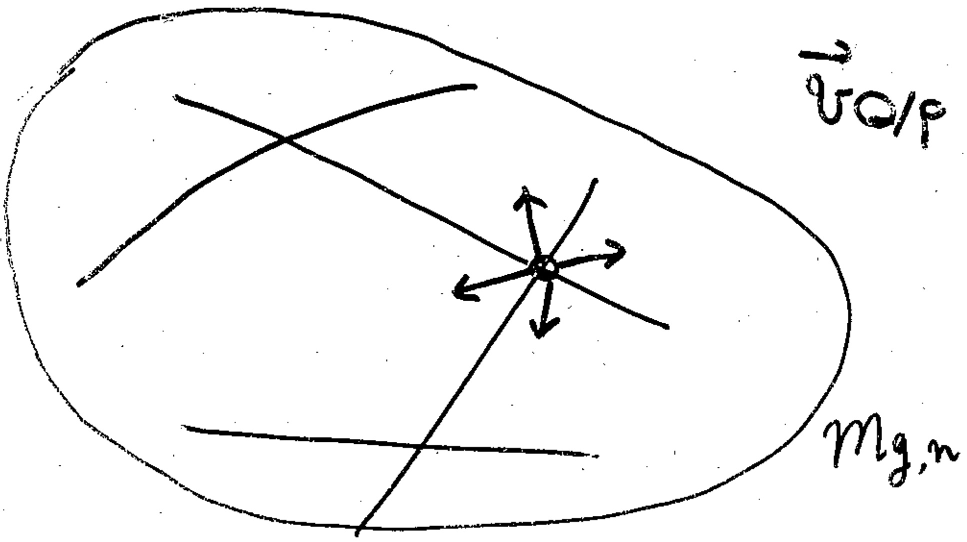
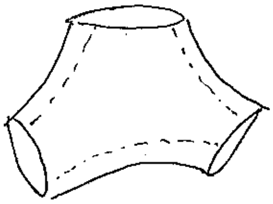
集中講義最終日スライド（中村博昭 2004.6月）

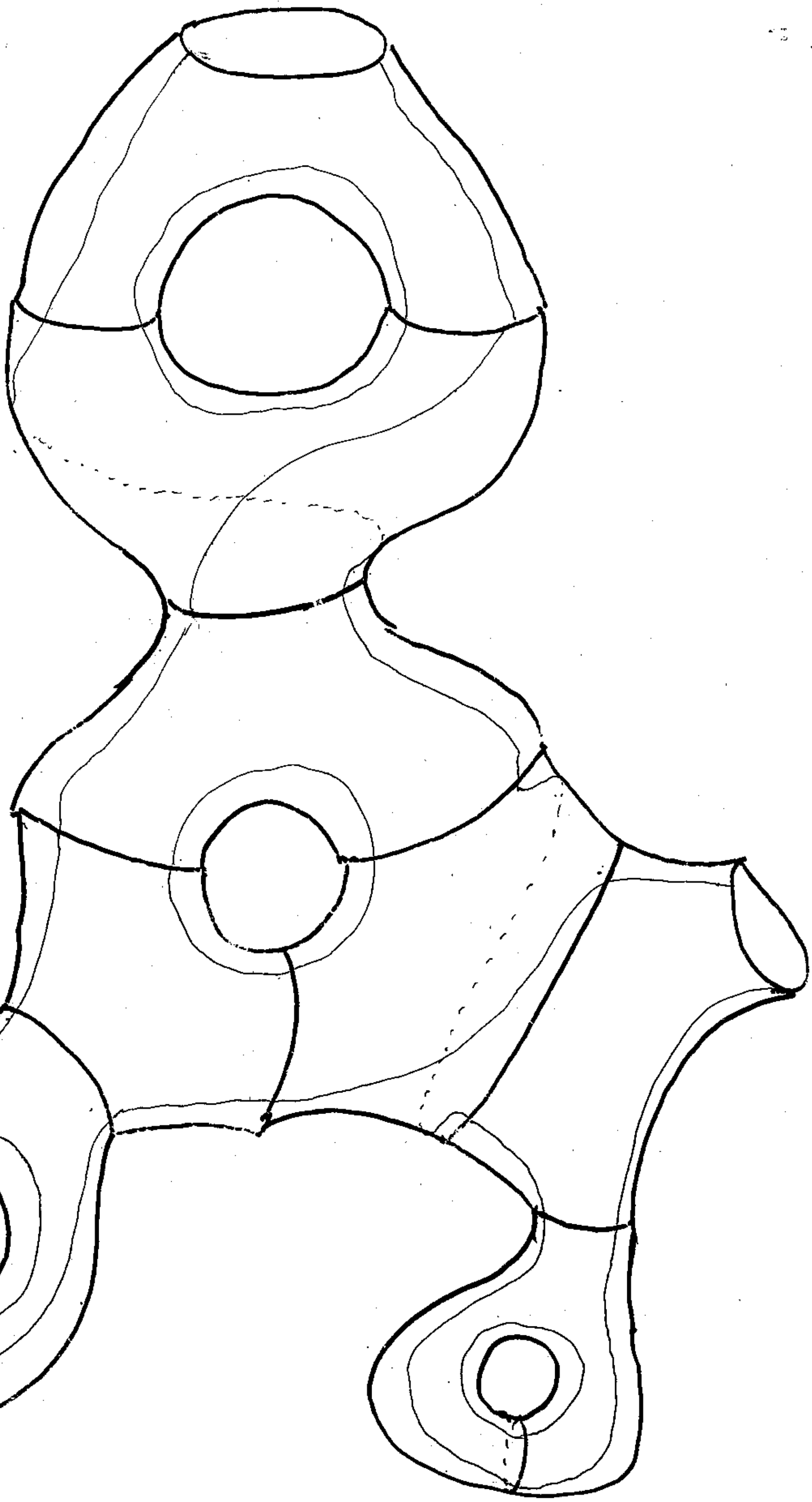
topology		algebraic geometry
pants-decomposition P	$\longleftrightarrow$	cusp of $\mathcal{M}_{g,n}$ (max. degeneration of stable marked curve)
quilt Q/P	$\longleftrightarrow$	tangential base point ("unit tangent vector" at cusp) $\vec{v}_{Q/P}$ on $M_{g,n}$ [Ihara-N]



Quilt / pants decomp.

Q/P on $\Sigma_{4,2}$





$$GT = \left\{ (\lambda, f) \in \hat{\mathbb{Z}}^{\times} \times \hat{F}_2 \;\middle|\; (\mathrm{I}), (\mathrm{II}), (\mathrm{III}) \right\}^{\times}$$

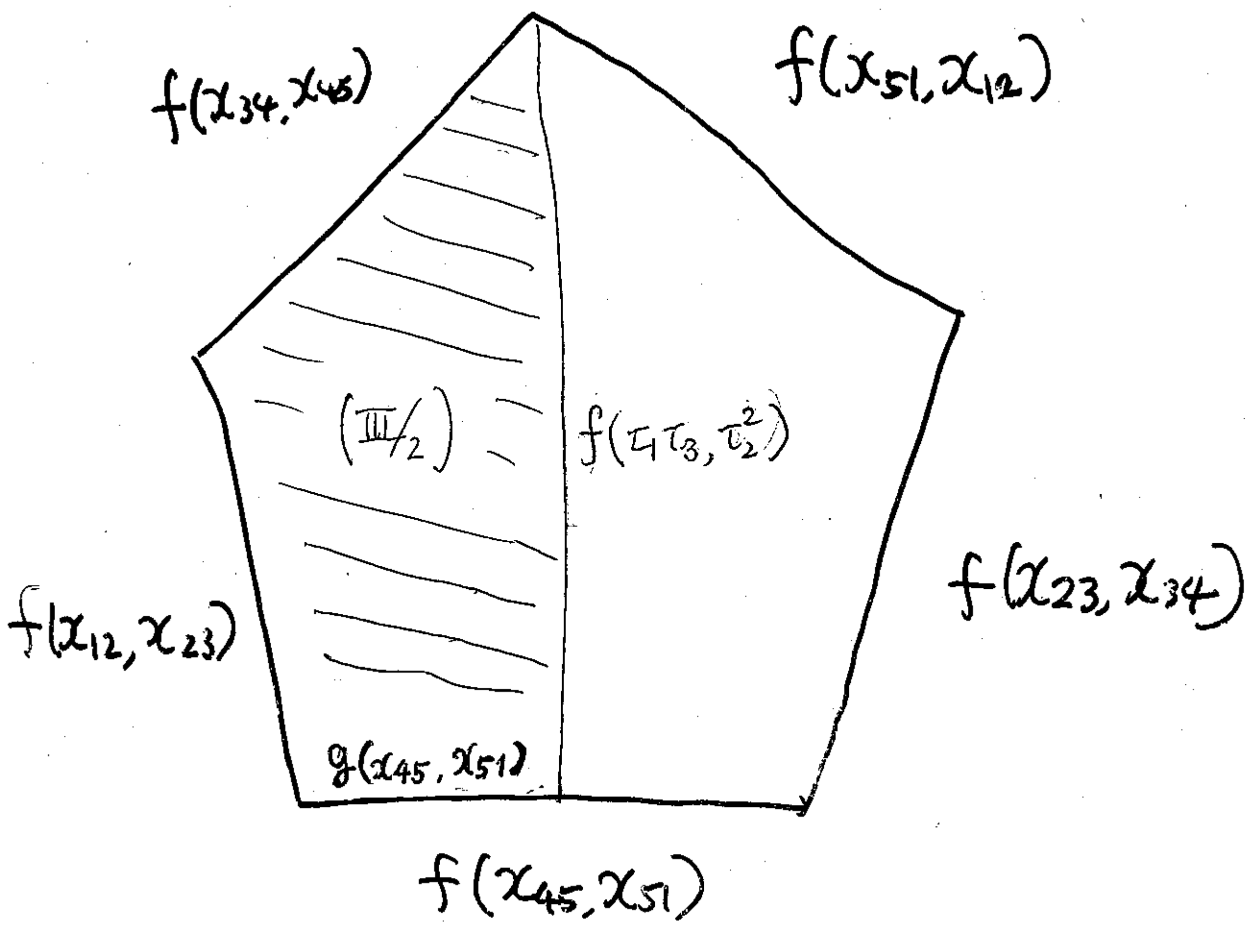
(I) $\quad f(x,y) = f(y,x)^{-1}$

(II) $\quad f(x,y)\, x^{\mu}\, f(y,x)\, y^{\mu}\, f(z,x)\, z^{\mu} = 1$

$$\mu = \frac{\lambda - 1}{2}, \qquad z = (xy)^{-1}$$

(III)

$$f(x_{12}, x_{23})\, f(x_{34}, x_{45})\, f(x_{51}, x_{12})\, f(x_{23}, x_{34})\, f(x_{45}, x_{51}) = 1$$



Refined Version of GT = (I), (II), (III/2), (IV)

(IV) $f(\tau_1, \tau_2^4) = \tau_2^{8\rho_2} f(\tau_1^2, \tau_2^2)\, \tau_1^{4\rho_2} (\tau_1\tau_2)^{-6\rho_2}$

here, $\hat{B}_3 = \langle \tau_1, \tau_2 \mid \tau_1\tau_2\tau_1 = \tau_2\tau_1\tau_2 \rangle$

$\rho_2 : \widehat{GT} \to \hat{\mathbb{Z}}$

1-cocycle extending Kummer-cycle

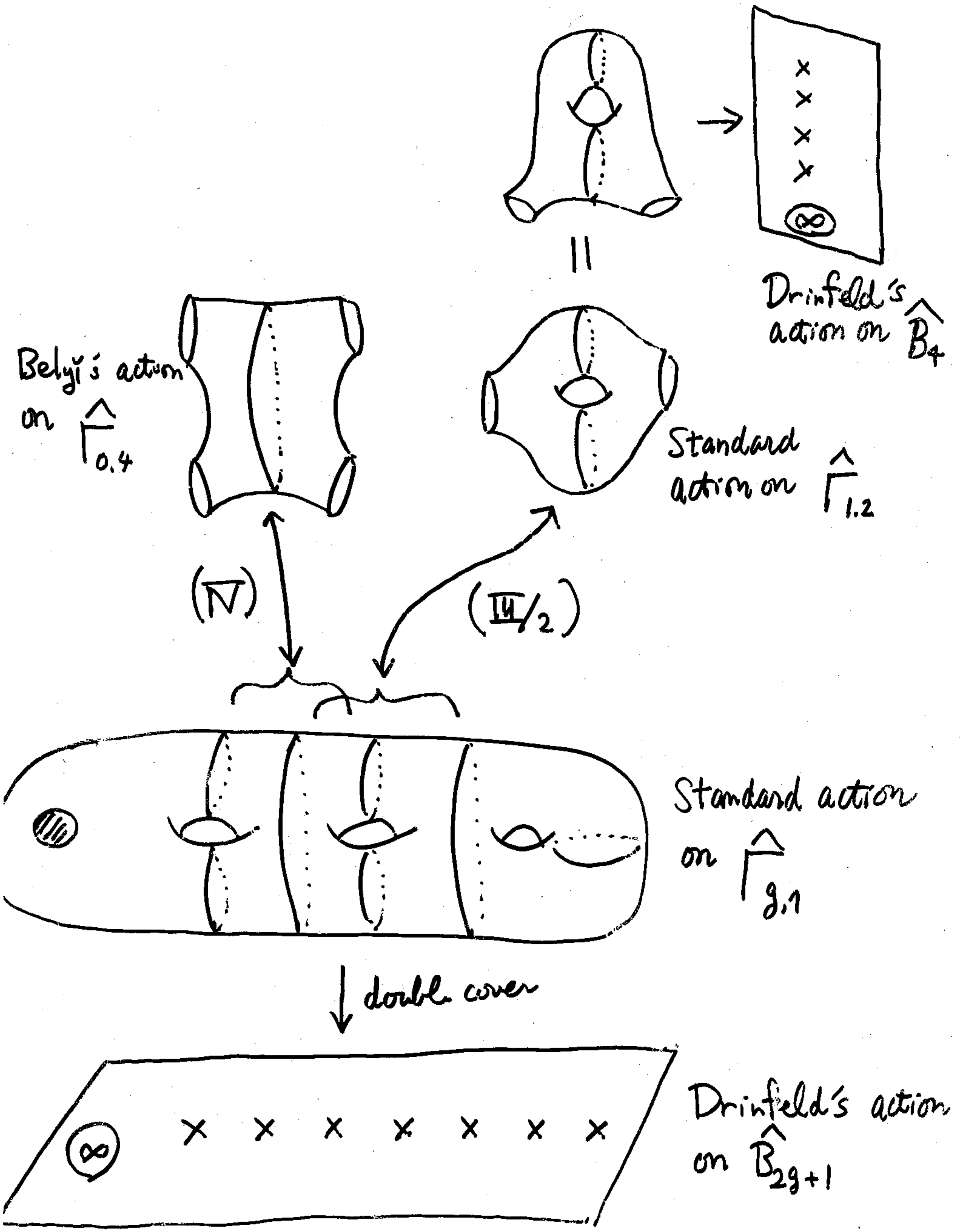
w.r.t. $\sqrt[n]{2}$

(III/2) $f(\tau_1\tau_3, \tau_2^2)$

$= g(x_{45}, x_{51})\, f(x_{12}, x_{23})\, f(x_{34}, x_{45})$

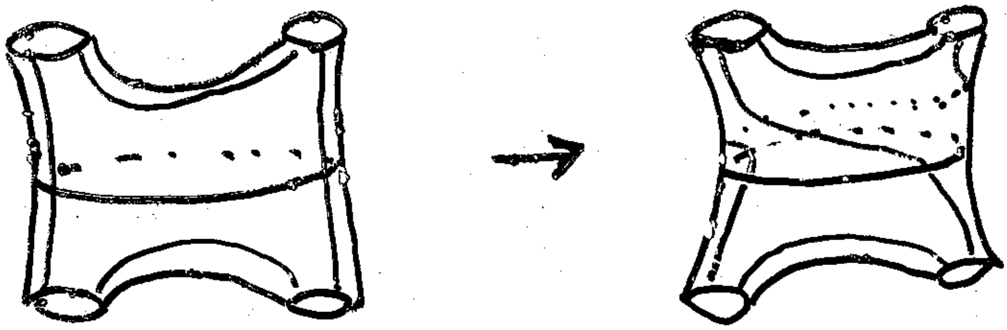
here $g(X, Y)$ is the unique elt $\in \hat{F}_2$

s.t. $f(X, Y) = g(Y, X)^{-1} g(X, Y)$

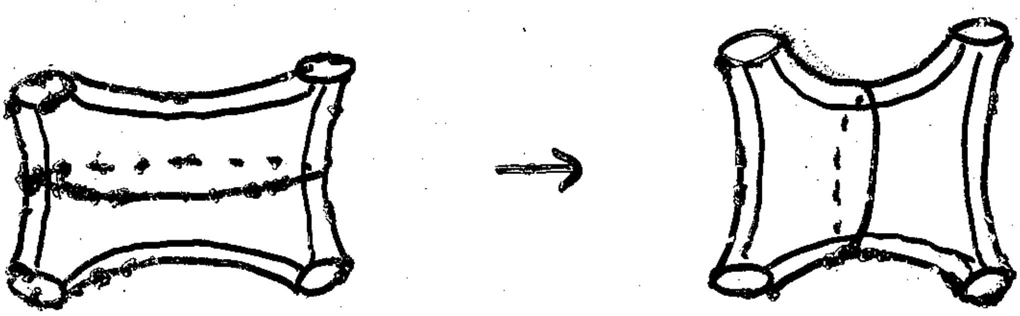


Drinfeld's action on $\widehat{B}_4$
Belyi's action on $\widehat{\Gamma}_{0,4}$
Standard action on $\widehat{\Gamma}_{1,2}$
(IV)
(III/2)
Standard action on $\widehat{\Gamma}_{g,1}$
double cover
Drinfeld's action on $\widehat{B}_{2g+1}$

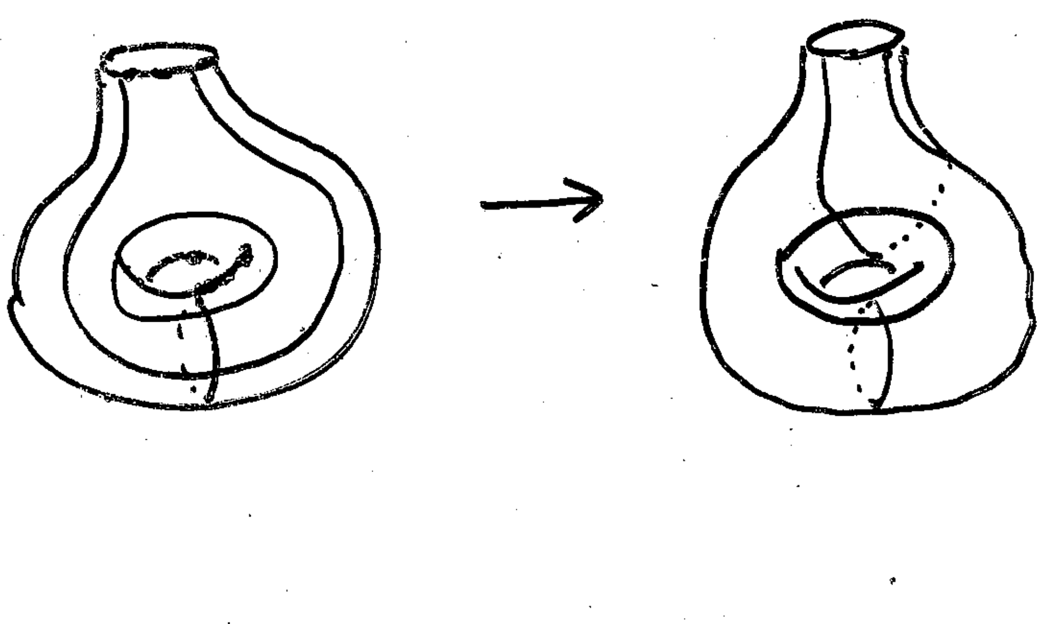
· half-twist



· A-move



· S-move



Generators of $\pi_1(M_{g,n};\{\vec{v}_{Q/p}\})$

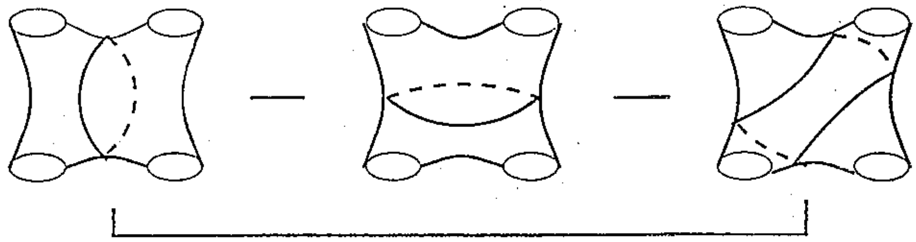
- half-twisting $\quad m(Q/p \to Q'/p) = m_T$
- A-move $\quad m(Q/p \to Q/p') = m_A$
- S-move $\quad m(Q/p \to Q'p') = m_S$

G_Q-actions

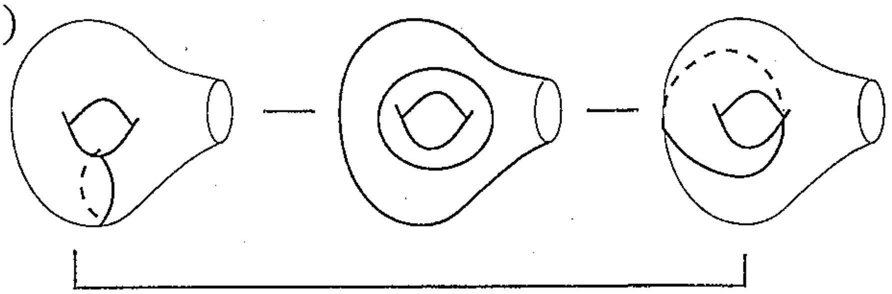
$$m = m_T, m_A, m_S \qquad (c \mapsto c')$$

$$\Rightarrow \begin{cases} \sigma(m_T) = D_c^{\frac{\lambda_\sigma - 1}{2}} \cdot m_T \\ \sigma(m_A) = f_\sigma(D_{c'}, D_c) \cdot m_A \\ \sigma(m_S) \\ \quad = D_c^{-8\rho_2(\sigma)} f_\sigma(D_{c'}^2, D_c^2) D_{c'}^{8\rho_2(\sigma)} (D_c D_{c'} D_c)^{\lambda_\sigma - 1} \cdot m_{S'} \end{cases}$$

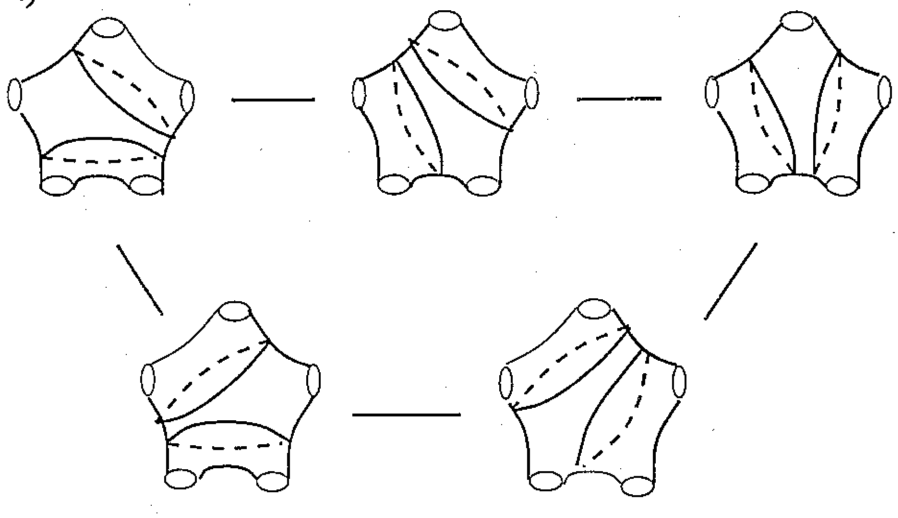
(3A)



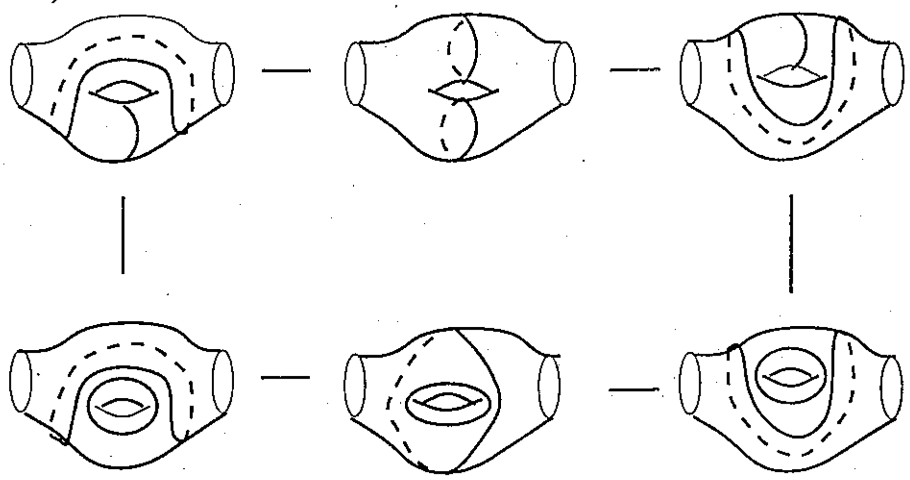
(3S)



(5A)



(6AS)



(c)