Corrigendum to
"Nonlinear geometric analysis on Finsler manifolds"
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Abstract

We list corrigenda to the survey article [4], due to the revisions of the articles [2, 3] whose results were reviewed in [4].

First, related to [2], we have the following corrigenda concerning the noncompact case.

- In the integrated form of the Bochner inequality [4, Theorem 3.5], we need to employ $u \in H^2_{\mathrm{loc}}(M) \cap \mathcal{C}^1(M)$ such that $\boldsymbol{\Delta} u \in H^1_{\mathrm{loc}}(M)$ and $\phi \in H^1_c(M) \cap L^\infty(M)$ as in the original paper [6, Theorem 3.6] (see also [2, Theorem 2.13], [5, Theorem 12.13]). We consider u and ϕ in the same classes also in the improved Bochner inequality [4, Proposition 4.4] (see [2, Corollary 3.6], [5, Corollary 12.16]).

- Among the properties of linearized heat semigroups in [4, Proposition 4.1(ii)], the Hölder continuity holds true and is enough for our applications. We refer to [2, Proposition 3.1] or [5, Proposition 13.20] for details.

- Due to the restriction of the class of admissible functions in the integrated Bochner inequality as above, we need some additional assumptions in the gradient estimates [4, Theorems 4.3, 4.5]. Precisely, we assume that $(M, F, \mathfrak{m})$ satisfies $\mathrm{Ric}_\infty \ge K$, $\mathsf{C}_F < \infty$ and $\mathsf{S}_F < \infty$, and that $(u_t)_{t \ge 0}$ is a global solution to the heat equation satisfying $u_0 \in \mathcal{C}^\infty_c(M)$ and

$$d[F(\boldsymbol{\nabla} u_t)]\big(\nabla^{\boldsymbol{\nabla} u_t}[F(\boldsymbol{\nabla} u_t)]\big) \in L^1(M) \tag{1}$$

  for all $t > 0$ (see [2, Theorem 3.7, Corollary 3.8]). We refer to [2, §3.4] for a further discussion on the (likely redundant) assumption (1).

- Similarly, in the characterizations of $\mathrm{Ric}_\infty \ge K$ in [4, Theorem 4.6] as well as the isoperimetric inequality [4, Theorem 6.3], we need to assume $\mathsf{C}_F < \infty$, $\mathsf{S}_F < \infty$, and (1) for all solutions $(u_t)_{t \ge 0}$ to the heat equation with $u_0 \in \mathcal{C}^\infty_c(M)$ (see [2, Theorems 3.9, 4.1]).

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Next, in the Sobolev inequality [4, Theorem 5.11] from [3], as explained in the erratum of [3], the case of $1 \le p < 2$ should be discussed separately and called the *Beckner inequality*. The statement itself of [4, Theorem 5.11] is true and, moreover, one can also show the following generalization to $N < 0$ along the lines of [1] (see [5, Theorem 16.8] for details).

Theorem 1 *Assume that* $(M, F, \mathfrak{m})$ *is compact and satisfies* $\mathrm{Ric}_N \ge K > 0$ *for some* $N \in (-\infty, -2)$ *and* $\mathfrak{m}(M) = 1$*. Then we have*

$$\frac{\|f\|_{L^p}^2 - \|f\|_{L^2}^2}{p-2} \le \frac{N-1}{KN} \int_M F^2(\boldsymbol{\nabla} f)\, d\mathfrak{m}$$

for all $1 \le p \le (2N^2+1)/(N-1)^2$ *and* $f \in H^1(M)$*.*

References

[1] I. Gentil and S. Zugmeyer, A family of Beckner inequalities under various curvature-dimension conditions. Bernoulli **27** (2021), 751–771.

[2] S. Ohta, A semigroup approach to Finsler geometry: Bakry–Ledoux's isoperimetric inequality. Comm. Anal. Geom. (to appear). Available at arXiv:1602.00390

[3] S. Ohta, Some functional inequalities on non-reversible Finsler manifolds. Proc. Indian Acad. Sci. Math. Sci. **127** (2017), 833–855; Erratum in *ibid.* **131** (2021), Paper No. 23, 2pp.

[4] S. Ohta, Nonlinear geometric analysis on Finsler manifolds. Eur. J. Math. **3** (2017), 916–952.

[5] S. Ohta, Comparison Finsler geometry. Springer Monographs in Mathematics. Springer, Cham, 2021.

[6] S. Ohta and K.-T. Sturm, Bochner–Weitzenböck formula and Li–Yau estimates on Finsler manifolds. Adv. Math. **252** (2014), 429–448.